Fluid Systems

(Understanding Engineering Thermo—Octave Levenspiel)

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Two points I want to emphasis:

1. **ΔU and ΔH**
2. **$W = W_{pv} + W_{sh}$**

BATCH OF IDEAL GAS

Batch of ideal gas

- Constant volume

$$v_1 = v_2 \quad and \quad \frac{p_1}{T_1} = \frac{p_2}{T_2}$$

$$w_{rev} = \int p dv = 0$$

$$q_{rev} = \Delta u + w_{rev} = c_v \Delta T + 0 = c_v \Delta T \quad \text{J/mol}$$

- Constant pressure

$$p_1 = p_2 \quad and \quad \frac{v_1}{T_1} = \frac{v_2}{T_2}$$

$$w_{rev} = \int p dv = p(v_2 - v_1) = p_1 v_1 \left(\frac{T_2}{T_1} - 1 \right) = \frac{p_1 v_1}{T_1}(T_2 - T_1) = R\Delta T$$

$$q_{rev} = \Delta u + w_{rev} = c_v \Delta T + R\Delta T = c_p \Delta T \quad \text{J/mol}$$

Batch of ideal gas

- Constant temperature

$$T_1 = T_2 \quad and \quad p_1 v_1 = p_2 v_2$$

$$\Delta h = \underbrace{\Delta u}_{0} + \underbrace{\Delta (pv)}_{0} = 0$$

$$q_{rev} = w_{rev} = \int p dv = \int \frac{RT}{v} dv$$

$$= RT \ln \frac{v_2}{v_1} = RT \ln \frac{p_1}{p_2}$$

J/mol

Batch of ideal gas

- Adiabatic (q=0) reversible process with constant c_v

$$\underbrace{du}_{c_v dT} = \overbrace{dq_{rev}}^{0} - dw_{rev} = \underbrace{-pdv}_{\frac{RT}{v} dv}$$

- Integrate

$$\int_{T_1}^{T_2} \frac{dT}{T} = -\frac{R}{c_v} \int_{v_1}^{v_2} \frac{dv}{v}$$

- Assume constant c_v, hence constant c_p,
- Introduce symbol k,

$$k = \frac{c_p}{c_v} = 1 + \frac{R}{c_v}$$

- On integration,

$$\ln\frac{T_2}{T_1} = -(k-1)\ln\frac{v_2}{v_1}$$

- therefore

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{k-1}$$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(k-1)/k}$$

$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{k}, \quad or \quad pv^{k} = \text{const.}$$

- For ideal adiabatic reversible batch process, $\Delta e_p = \Delta e_k = 0$, $q_{rev} = 0$,

$$w_{rev} = \int p dv = \int_{v_1}^{v_2} \frac{const.}{v^k} dv$$

$$pv^k = \text{const.}$$

$$= \frac{p_1 v_1}{k-1}\left[1 - \left(\frac{p_2}{p_1}\right)^{(k-1)/k}\right] = \frac{RT_1}{k-1}\left[1 - \left(\frac{p_2}{p_1}\right)^{(k-1)/k}\right]$$

$$w_{rev} = -\Delta u = c_v (T_2 - T_1)$$

$$= -\frac{R}{k-1}(T_2 - T_1)$$

$$k = \frac{c_p}{c_v} = 1 + \frac{R}{c_v}$$

$$= -\frac{p_2 v_2 - p_1 v_1}{k-1}$$

$$pv = RT$$

Example I

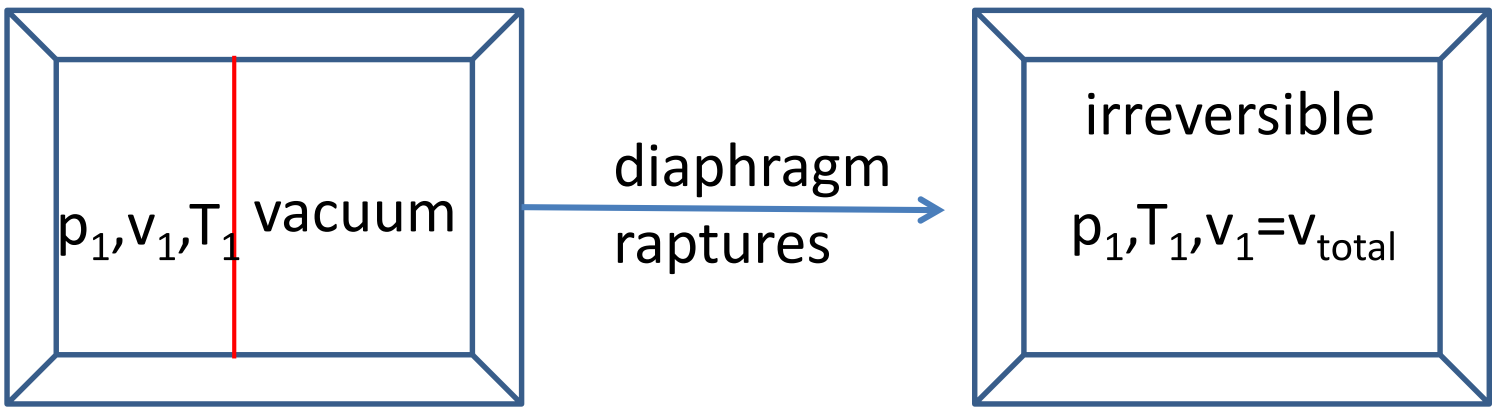
- Slow leak from an insulated tank

  The gas remaining in the tank experiences and adiabatic reversible expansion, therefore

$$\boxed{\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(k-1)/k}}$$

Example II

- Rupture of a diaphragm in an insulated tank



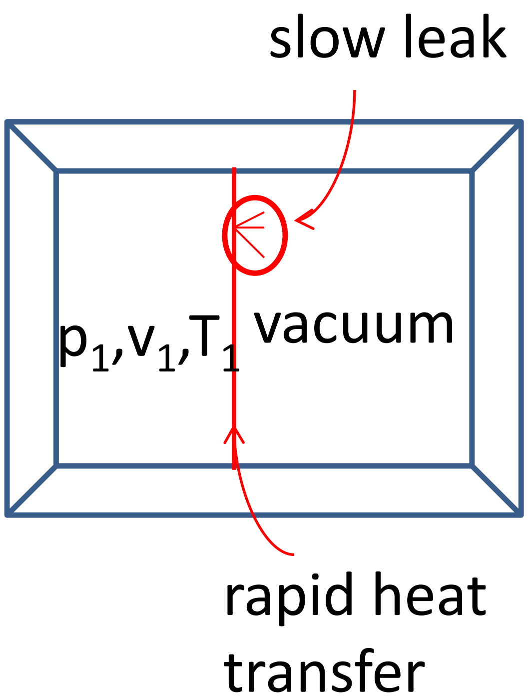
- Insulated and const volume

$$\Delta U = \overset{0}{\cancel{Q}} - \overset{0}{\cancel{W}}$$

$$T_2 = T_1 \quad and \quad \frac{p_2}{p_1} = \frac{V_2}{V_1}$$

Example III

- Slow leak between sections of an insulated tank



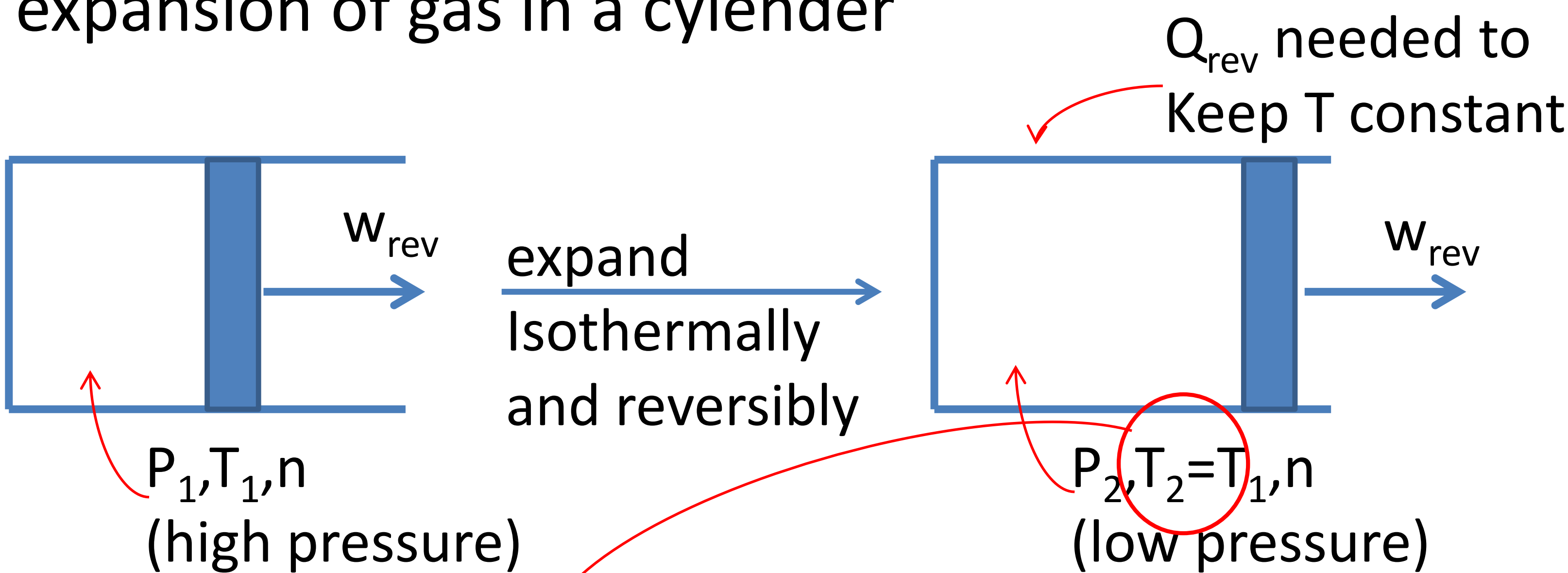
Insulated and const volume

$$\Delta U = \overset{0}{\cancel{Q}} - \overset{0}{\cancel{W}}$$

$$T_2 = T_1 \quad and \quad \frac{p_2}{p_1} = \frac{V_2}{V_1}$$

Example IV

- Heat involved in the slow isothermal reversible expansion of gas in a cylender



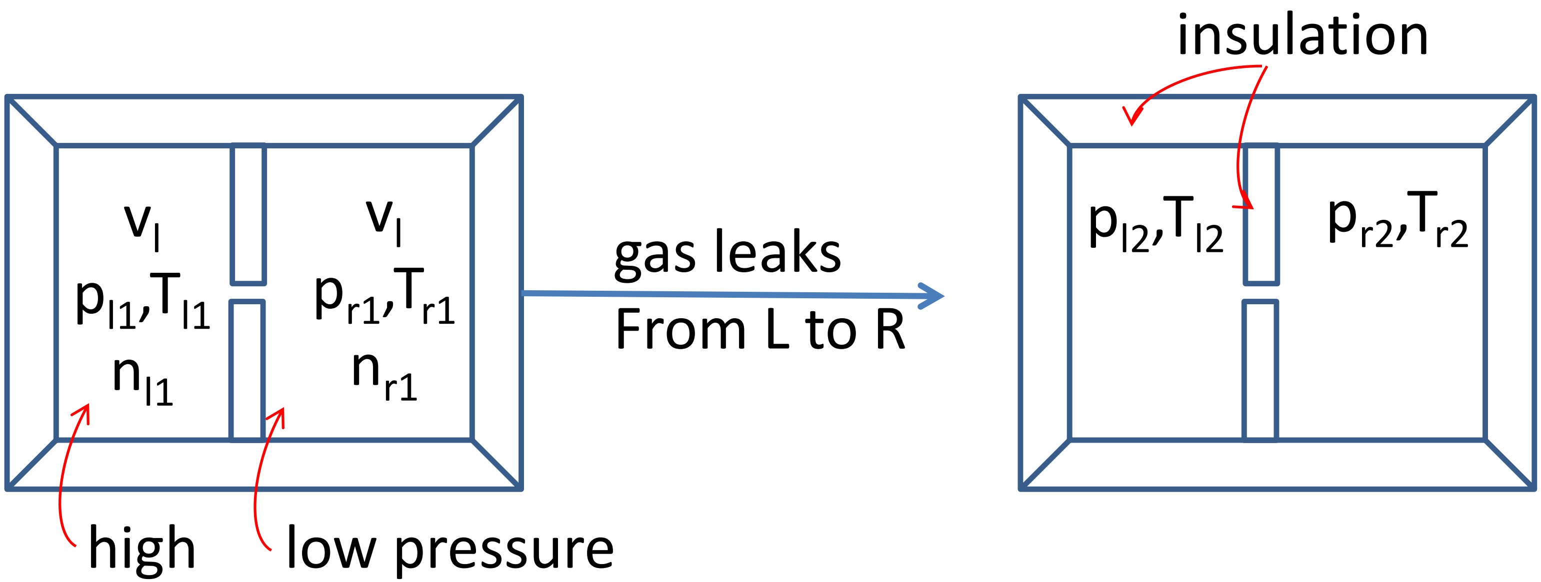
$$0 = \Delta U = Q_{rev} - W_{rev}$$

$$Q_{rev} = W_{rev} = nRT \ln \frac{p_1}{p_2}$$

constant temperature process

Example V

- Leak between two interconnected insulated tanks



- The gas remaining on the left-hand side expands adiabatically and reversibly.
- Find how T_l, T_r, p_l and p_r change

$$n_{l1} = \left(\frac{pv}{RT}\right)_{l1}$$

$$n_{r1} = \left(\frac{pv}{RT}\right)_{r1}$$

$n_{l1}+n_{r1} = n_{total}$

$$n_{l2} = \left(\frac{pv}{RT}\right)_{l2}$$

$$n_{r2} = \left(\frac{pv}{RT}\right)_{r2}$$

$n_{l2}+n_{r2} = n_{total}$

adiabatically and reversibly expansion,

$$\boxed{\frac{T_{l2}}{T_{l1}} = \left(\frac{p_{l2}}{p_{l1}}\right)^{(k-1)/k}}$$

$$\sum E_2 = \sum E_1 : (n_l u_l)_2 + (n_r u_r)_2 = (n_l u_l)_1 + (n_r u_r)_1$$

$$\boxed{(n_l T_l)_2 + (n_r T_r)_2 = (n_l T_l)_1 + (n_r T_r)_1}$$

$$\boxed{\begin{array}{l} u = c_v T \\ h = c_p T \end{array}}$$

Example VI

- Total work done by an expanding gas

A 2 liter plastic pop bottle contains air at 300K and 1.5 bar gauge pressure. How much work could be done by this gas if you could expand it down to 1 bar

- – Isothermally and reversibly?
- – Adiabatically and reversibly?

$$n = \frac{pv}{RT} = \frac{(12.5 \times 15^{5})(0.002)}{(8.314)(300)} = 1.00 \text{ mol}$$

- Isothermal expansion

$$\boxed{W_{rev} = nRT \ln \frac{p_1}{p_2}}$$

$$W_{rev} = (1)(8.314)(300) \ln \frac{12.5}{1} = 6300 \text{ J}$$

- Adiabatic expansion

$$\boxed{W_{rev} = \frac{nRT_1}{k-1} \left[1 - \left(\frac{p_2}{p_1} \right)^{(k-1)/k} \right]}$$

$$W_{rev} = \frac{(1)(8.314)(300)}{1.4-1} \left[1 - \left(\frac{1}{12.5} \right)^{0.4/1.4} \right] = 3205 \text{ J}$$

Example VII

- Net work done by an expanding gas

The previous example calculated the work done by an expanding gas. However, in doing so the gas had to push back the 1 bar atmosphere. Let us now account for this work, subtract it from the work done, and thereby evaluate the useful work (shaft work) that could be extracted by this

  - Isothermal expansion
  - Adiabatic expansion

- Isothermal expansion
  - The work needed to push back the atmosphere is

$$W_{pv} = p_0(v_2 - v_1)$$

$$= (1 \times 10^5)(12.5 \times 0.002 - 0.002) = 2300 \text{ J}$$

  - The reversible shaft work that can be extracted is

$$\boxed{W_{sh} = 6300 - 2300 = 4000 \text{ J}}$$

- Adiabatic expansion
  - Final temperature of the expanded air is not 300K, but

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{(k-1)/k} = 300\left(\frac{1}{12.5}\right)^{0.4/1.4} = 146 \text{ K}$$

$$W_{pv} = p_0(v_2 - v_1)$$

$$= (1 \times 10^5)(12.5 \frac{146}{300} 0.002 - 0.002) = 1015 \text{ J}$$

  - The reversible shaft work that can be extracted is

$$\boxed{W_{sh} = 3205 - 1015 = 2190 \text{ J}}$$

Example VIII

- Explosion: The popping pop bottle
  - reversible or irreversible?
  - Isothermal or adiabatic?
- From the first law, and adiabatic

$$\Delta U = \cancelto{0}{Q} - W_1$$

- Highly irreversible, all go to push back

$$W_2 = \int p\,dV = p_{surr}\,\Delta V$$

- Therefore,

$$W_1 = \Delta U = nc_v (T_{initial} - T_{final})$$

$$W_2 = p_{surr} (v_{finall} - v_{initial})$$

- Then

$$W_1 = (1)(29.099 - 8.314)(300 - T_{final})$$

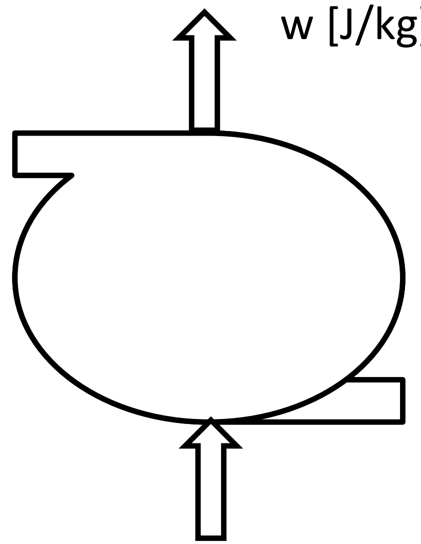
$$W_2 = 1 \times 10^5 \left[12.5 \times 0.002 \left(\frac{T_{final}}{300} \right) - 0.002 \right]$$

- Solving T_{final} by equating $W_1 = W_2$
- Finally

$$\boxed{\begin{array}{l} T_{finall} = 221 \text{ K} \\ \text{W} = 1642 \text{ J} \end{array}}$$

STEADY STATE FLOW SYSTEMS

Steady State Flow System



$e_1=u_1+e_{p1}+e_{k1}$ [J/kg]

$$\left(w_{pv,in} = \int_0^{v_1} p\,dv = p_1 v_1 \right)$$

$e_2=u_2+e_{p2}+e_{k2}$ [J/kg]

$$\left(w_{pv,out} = \int_0^{v_2} p\,dv = p_2 v_2 \right)$$

$$\underbrace{(\Delta U+\Delta E_p+\Delta E_k)}_{\Delta E} = Q - W$$

work done by system

$$w_{total} = w_{sh} + \underbrace{\int_0^{v_2} p\,dv}_{= p_2 v_2} - \underbrace{\int_0^{v_1} p\,dv}_{= p_1 v_1}$$

the rest work done
by the system—shaft work

Steady State Flow System

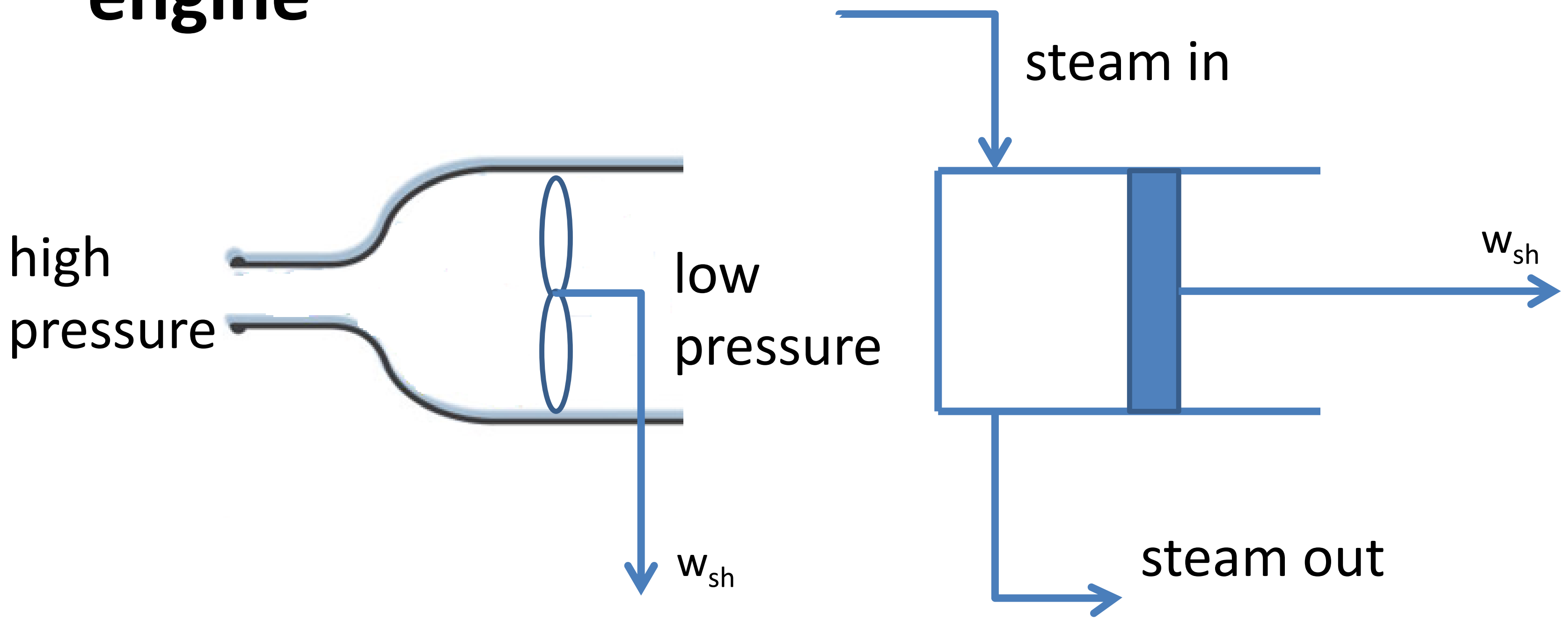
- **Rearrange**

$$\underbrace{\underbrace{(u_2 + p_2 v_2)}_{h_2} - \underbrace{(u_1 + p_1 v_1)}_{h_1}}_{\Delta h} + \overbrace{g\Delta z + \frac{1}{2}\Delta v^2}^{0} = q - w_{sh}$$

for the flow streams,
not the system

Example I

- **The steam or water turbine and the steam engine**

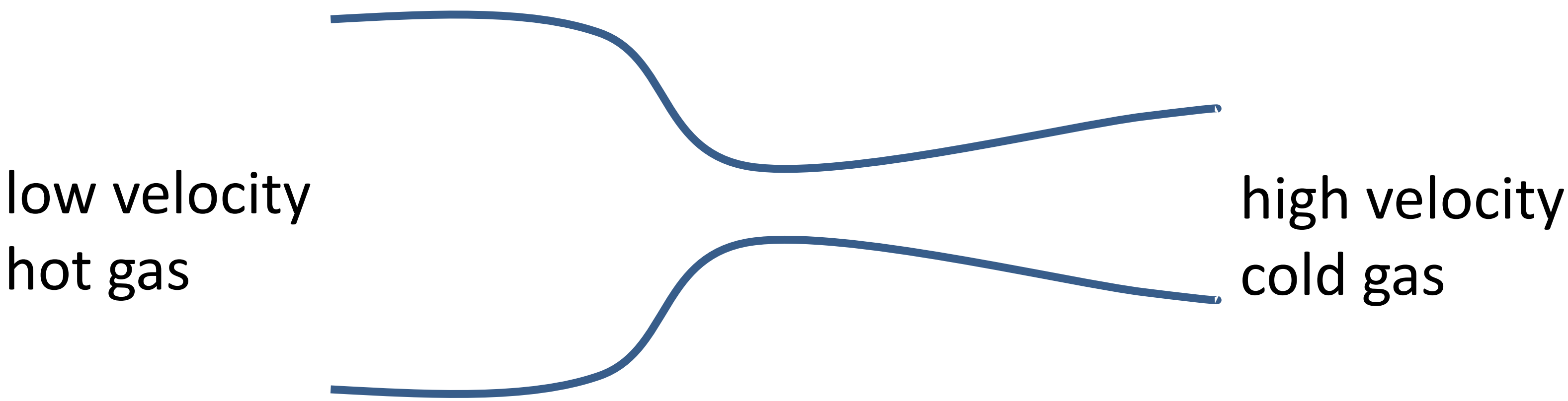


$$\dot{m}\Delta h = \dot{m}(h_2 - h_1) = -\dot{w}_{sh} \quad [W]$$

$$\Delta h = -w_{sh} \quad [J/kg]$$

Example II

- **The adiabatic flow nozzle**

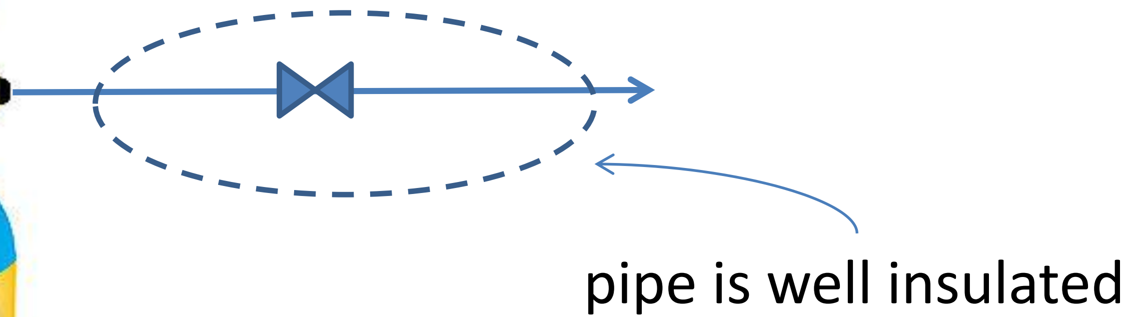


$$\dot{m}\Delta h = -\frac{\dot{m}}{2} V_2^2 \qquad [W]$$

$$\Delta h = -\frac{1}{2} V_2^2 \qquad [\frac{J}{kg}]$$

Example III

- **The Joule-Thomson expansion**

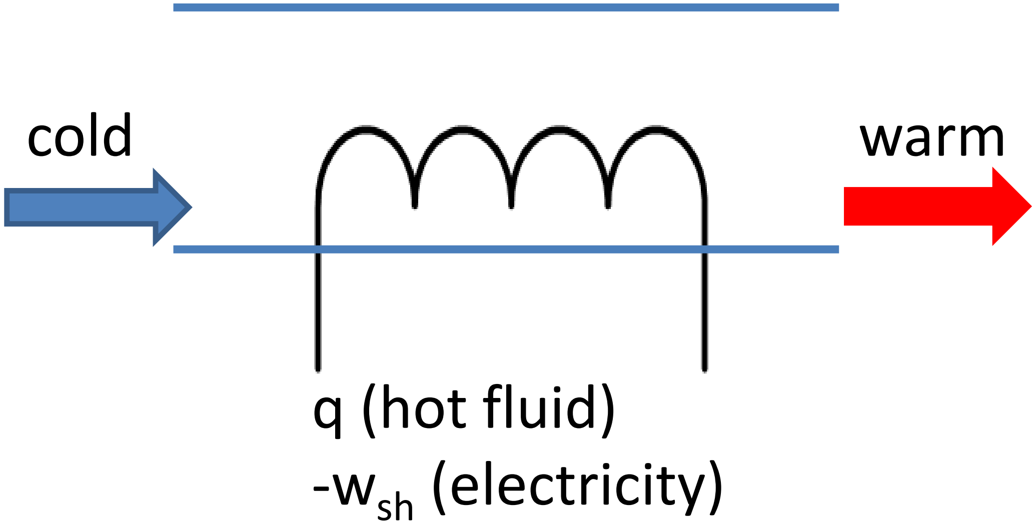


Across the slow leak or porous plug

$$\Delta h = h_2 - h_1 = 0$$

Example IV

- **The flow heater**



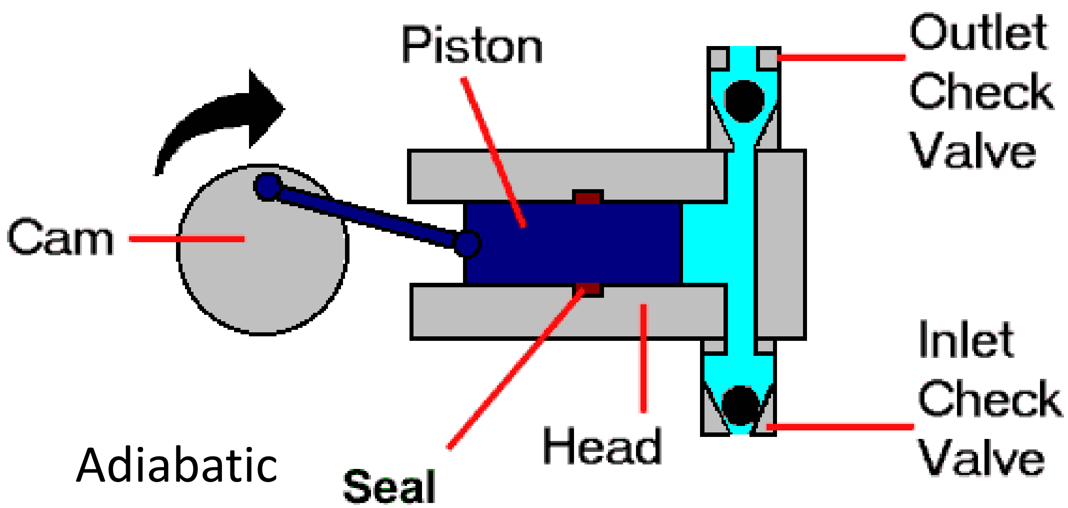
$$\dot{m}\Delta h = \dot{q}$$

$$\dot{m}\Delta h = -\dot{w}_{sh}$$

heat or work done on fluid in the heater

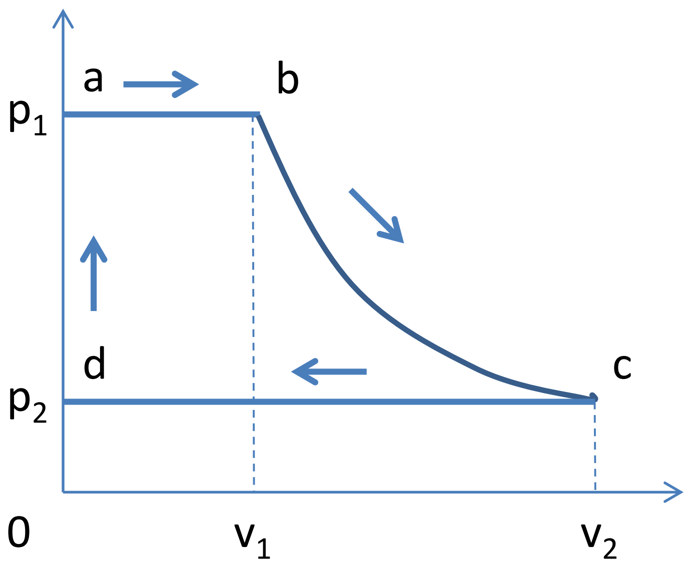
Example V

- **Ideal piston-cylinder engine or ideal piston-cylinder pump**



Adiabatic

Example V



- a-b Introduce 1 kg of high pressure gas at p_1 and of volume v_1.

$$w_1 = \int_0^{v_1} p_1 dv = p_1 v_1$$

- b-c Expand the gas to the outlet pressure p_2. (both valves are closed)

$$w_2 = \int_{v_1}^{v_2} p\, dv$$

- c-d Push out all the gas in the cylinder.

$$w_3 = \int_{v_2}^{0} p_2 dv = p_2 v_2$$

- Net shaft work done by the fluid

$$w_{sh} = w_1 + w_2 + w_3$$

$$= p_1 v_1 + \int_{v_1}^{v_2} pdv - p_2 v_2$$

- From the pv diagram

$$= -\int_{p_1}^{p_2} vdp$$

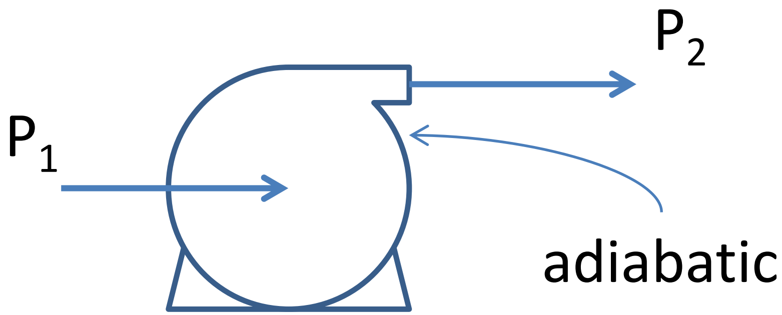
- Or $d(pv) = pdv + vdp$

$$\int_1^2 d(pv) = p_2 v_2 - p_1 v_1 = \int pdv + \int vdp$$

$$\Delta h = -w_{sh} = +\int_1^2 vdp \qquad [J / kg]$$

Example VI

- Ideal turbine or compressor



$$\Delta u + \Delta e_p + \Delta e_k = \cancel{q} - w = -\int p dv$$

$$\Delta u + \Delta e_p + \Delta e_k = -p_2 v_2 + p_1 v_1 + \int v dp$$

$$\Delta h + \Delta e_p + \Delta e_k = +\int v dp$$

- If constant v

$$\Delta h + \Delta e_p + \Delta e_k = v \Delta p = \frac{\Delta}{\rho}$$

Example VII

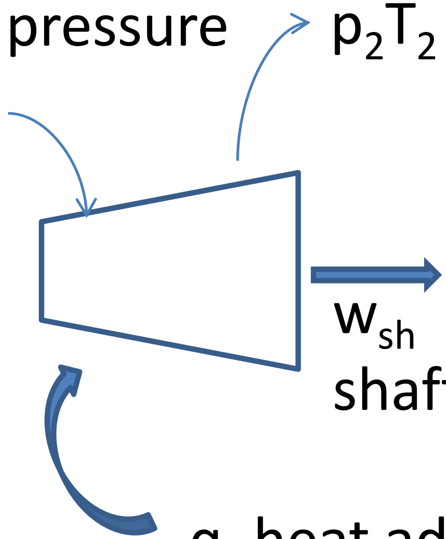
- Ideal isothermal work-producing machine

high pressure

p_1T_1

low pressure

$p_2T_2 = T_1$



w_{sh}

shaft work done

q, heat added

to keep $T_2 = T_1$

Assume $\Delta e_p = \Delta e_k = 0$

Noting that $T_1 = T_2$,

then $P_1v_1 = p_2v_2$

$\Delta h = 0$

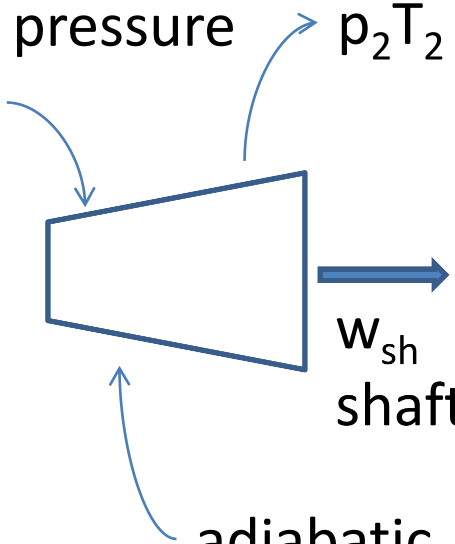
$$w_{sh} = q = -\int v\,dp = RT \ln \frac{p_1}{p_2}$$

Or

$$\dot{W}_{sh} = \dot{Q} = \dot{n} RT \ln \frac{p_1}{p_2}$$

Example VIII

- Ideal reversible work-producing machine



$$w_{flow} = w_{batch} - \Delta pv$$

$$w_{flow} = -\frac{\left(p_2v_2 - p_1v_1\right)}{k-1} - \left(p_2v_2 - p_1v_1\right)$$

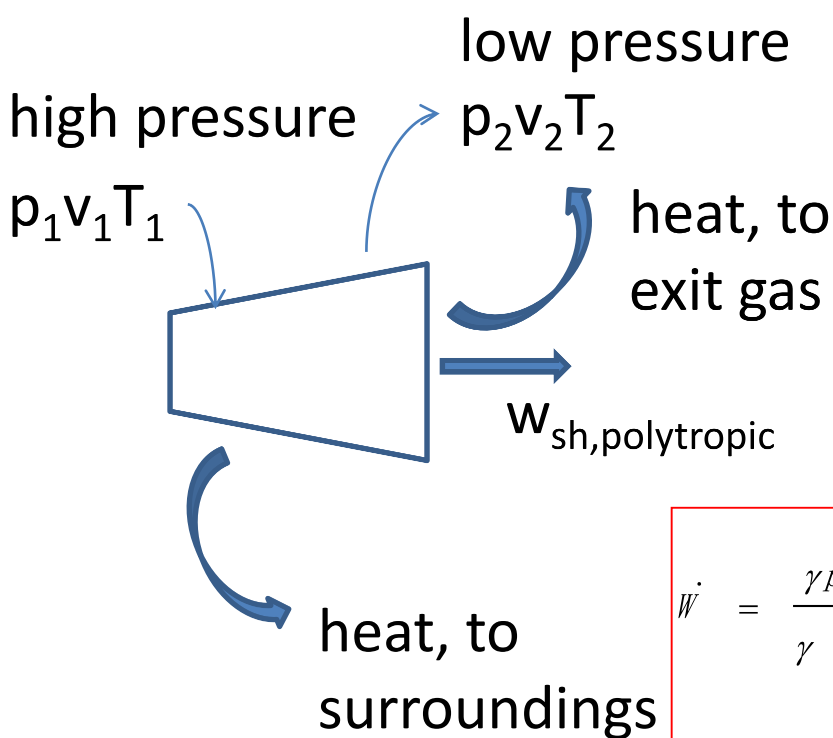
$$= -\frac{k}{k-1}\left(p_2v_2 - p_1v_1\right) = kw_{batch}$$

$$\dot{W} = \frac{kp_1\dot{V}_1}{k-1}\left[1 - \left(\frac{p_2}{p_1}\right)^{(k-1)/k}\right] = \dot{n}c_pT_1\left[1 - \left(\frac{p_2}{p_1}\right)^{(k-1)/k}\right]$$

$$= \dot{n}c_p\left(T_2 - T_1\right) = -\frac{k}{k-1}\left(p_2\dot{V}_2 - p_1\dot{V}_1\right)$$

Example IX

- Real turbines and compressors



$$w_{sh,poly} = \gamma w_{batch}$$

$$w_{flow} = -\frac{(p_2v_2 - p_1v_1)}{k-1} - (p_2v_2 - p_1v_1)$$

$$= -\frac{k}{k-1}(p_2v_2 - p_1v_1) = k w_{batch}$$

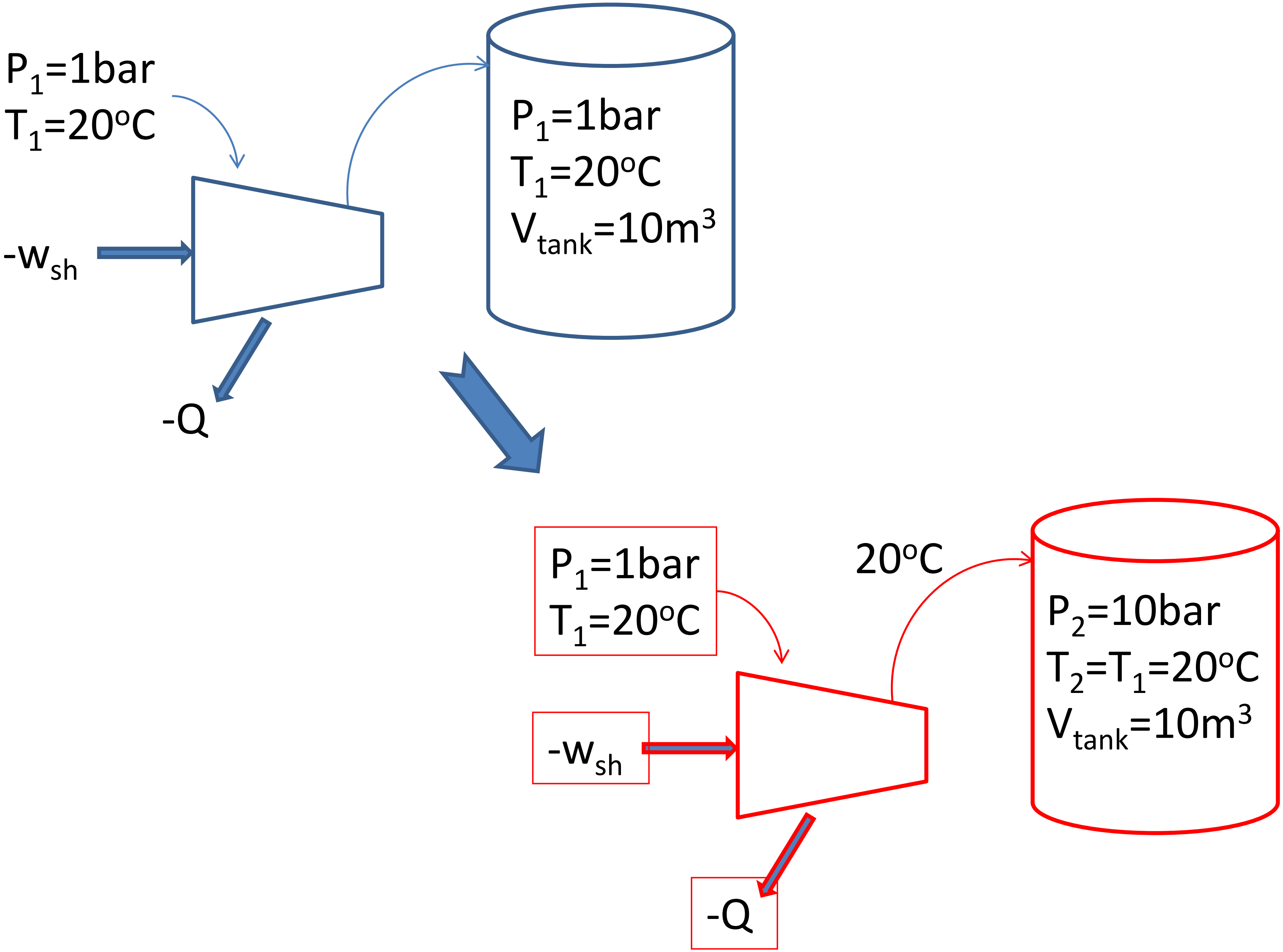
replace k with γ

$$\dot{W} = \frac{\gamma p_1 \dot{V}_1}{\gamma - 1}\left[1 - \left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma}\right] = \frac{\gamma \dot{n} R T_1}{\gamma - 1}\left[1 - \left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma}\right]$$

$$= \gamma \dot{n} c_p (T_2 - T_1) = -\frac{\gamma}{\gamma - 1}(p_2\dot{V}_2 - p_1\dot{V}_1)$$

Example X

- Pumping up a tank with an ideal gas
- A 10 m^3 tank is open to the surroundings at 20^oC and 1 bar. A compressor connected the tank pumps air into the tank. The compressor operates isothermally.
  - Find the minimum work required to pressurize the tank to 10 bar.
  - Find the heat interchange at the compressor.



P_1=1bar
T_1=20°C
P_1=1bar
T_1=20°C
V_{tank}=10m³
$-w_{sh}$
-Q
P_1=1bar
T_1=20°C
20°C
P_2=10bar
T_2=T_1=20°C
V_{tank}=10m³
$-w_{sh}$
-Q

- Recall from example VII

$$W_{sh} = RT_1 \ln \frac{p_1}{p_2}$$

$$dW_{sh} = RT_1 \ln \frac{p_1}{p} dn$$

- Ideal gas EOS

$$pv = nRT$$

$$n = \frac{V_{\text{tank}}\, p}{RT_1}$$

$$dn = \frac{V_{\text{tank}}}{RT_1} dp$$

- Recall from example VII

$$\boxed{dW_{sh} = V_{\text{tank}} \ln \frac{p_1}{p} dp}$$

- or

$$\boxed{W_{sh} = V_{\text{tank}} \int_{p_1}^{p_2} \ln \frac{p_1}{p} dp}$$

$$\boxed{W_{sh} = 10\, m^3 \int_{10^5}^{10^6} \ln \frac{10^5}{p} dp = -13.5 \times 10^6 \text{ J}}$$

$$m\left[\Delta h + \Delta e_p + \Delta e_k\right] = Q - W_{sh}$$

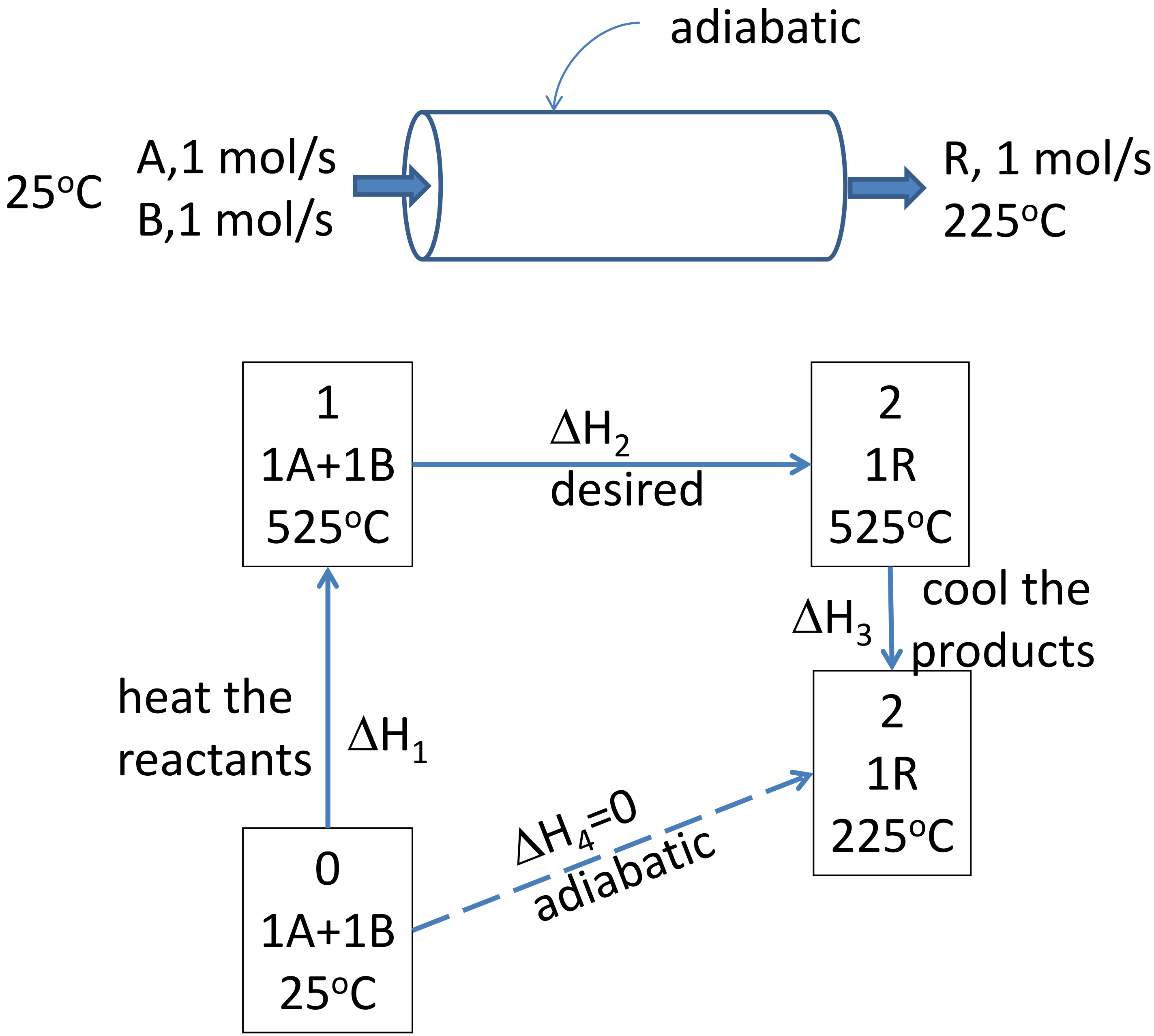
$$Q = W_{sh}$$

Example XI

- **The Flow Reactor** : 1 mol/s of gaseous A and 1 mol/s of gaseous B, both at 25^{o}C, are pumped continuously into an adiabatic mixer-reactor. They react to completion accordig to the stoichiometry.

$$A + B \rightarrow R$$

- The product stream, also gaseous, leaves the reactor at 225^{o}C. Find the ΔH_r for the above reaction at 525 ^{o}C.
- Data c_{pA} = 30, c_{pB} = 40, c_{pR} = 50 J/mol/K


adiabatic
25°C
A,1 mol/s
B,1 mol/s
R, 1 mol/s
225°C
1
1A+1B
525°C
ΔH2
desired
2
1R
525°C
cool the
products
ΔH3
heat the
reactants
ΔH1
2
1R
225°C
0
1A+1B
25°C
ΔH4=0
adiabatic

$$\Delta H + \overset{0}{\overline{(m\omega)}g\Delta z} + \overset{0}{\frac{\overline{(m\omega)}}{2}\Delta \mathbf{v}^2} = \overset{0}{Q} - \overset{0}{W_{sh}}$$

$$\Delta H_1 + \Delta H_2 + \Delta H_3 = \Delta H_4 = 0$$

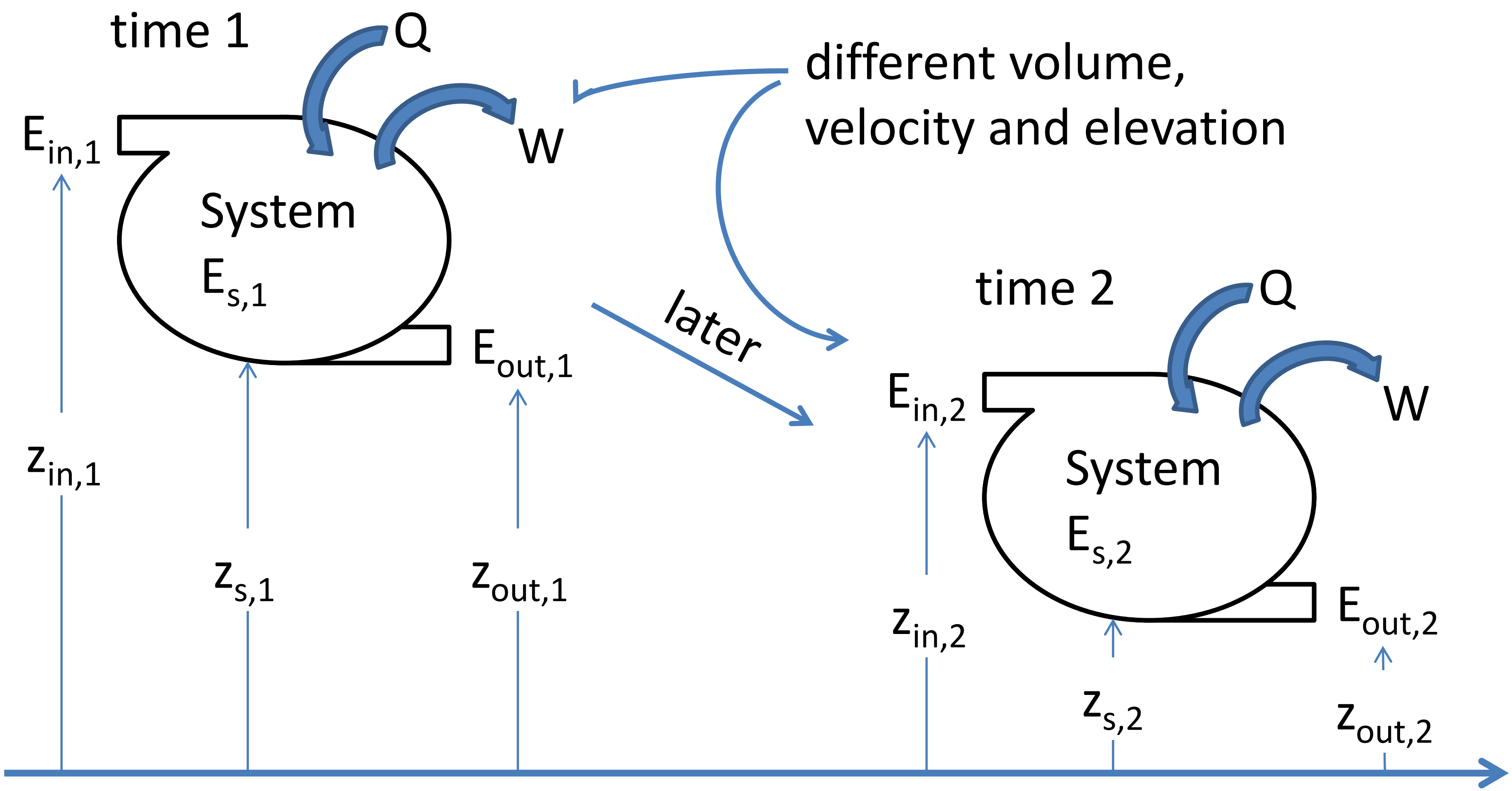
$$\Delta H_2 = -\Delta H_1 - \Delta H_3$$

$$= -\left[1 \cdot c_{pA}(T_1 - T_0) + 1 \cdot c_{pB}(T_1 - T_0)\right] - 1 \cdot c_{pR}(T_3 - T_2)$$

$$= -20 \ \mathrm{kJ/mol}$$

UNSTEADY STATE FLOW SYSTEMS

Unsteady state flow systems



$$\Delta \mathbf{E}_{system} = \text{(all energy inputs)} - \text{(all energy outputs)}$$

$$= - \Delta \mathbf{E}_{streams} + Q - W$$

$$\underbrace{m_2(u+e_p+e_k)_2 - m_1(u+e_p+e_k)_1}_{\textit{system}} + \underbrace{m_{out}(u+e_p+e_k)_{out} - m_{in}(u+e_p+e_k)_{in}}_{\textit{streams}}$$

$$= Q - W$$

where $W = W_{sh} + W_{pv,system} + W_{pv,streams}$

$$m_2(u+e_p+e_k)_2 - m_1(u+e_p+e_k)_1 + m_{out}(h+e_p+e_k)_{out} - m_{in}(h+e_p+e_k)_{in}$$

$$= Q - (W_{sh} + W_{pv,system})$$

$$= Q - \left(W_{sh} + \int_{V_1}^{V_2} p_{\,system}\, dv\right)$$

volume change in system

Example I

- Filling a glass with water

Hot water (80°C) from a kettle is poured into a completely insulated styrofoam cup. Apply the general equation to the cup to find the temperature of the water in the cup.

$$(m_{cup}u_{cup})_2 + (m_{water}u_{water})_2 - (m_{cup}u_{cup})_1 - \underbrace{m_{kettle}h_{kettle}}^{\boxed{\text{water 1}}} = Q - p_2v_2$$

assume $u_{cup1} = u_{cup2}$, $Q=0$, $p_2 = 1$ bar, v_2 = hot water

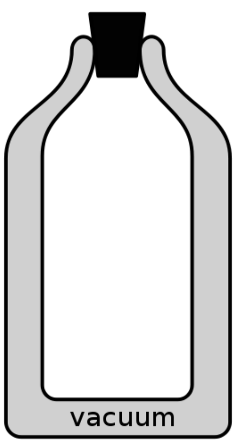
$$(m_{cup}\cancel{u_{cup}})_2 + (m_{water}u_{water})_2 - (m_{cup}\cancel{u_{cup}})_1 - m_{kettle}h_{kettle} = \cancel{Q} - p_2v_2$$

$u_{water2} - h_{kettle} = - p_2v_{water2}$ or $h_{water2} = h_{kettle}$, $T_{water2} = T_{kettle} = 80^oC$

Example II

- Filling an evacuated tank with an ideal gas

A valve on an evacuated insulated tank is opened. Air (an ideal gas) rushes in and the pressure equalizes. The valve is then quickly closed. What is the temperature of the gas in the tank if room temperature is 27°C and pressure is 1 bar.



$$m_2(u+\cancel{e_p}+\cancel{e_k})_2 - \overset{=m_2}{m_{in}}(h+\cancel{e_p}+\cancel{e_k})_{in} = \cancel{Q} - (\cancel{W_{sh} + W_{pv,system}})$$

$$u_2 = h_{in} \quad \text{or} \quad c_v T_2 = c_p T_{in}$$

$$T_2 = \left(\frac{29.1}{29.1 - 8.314}\right)(300) = 420\text{K} = 147\ ^\circ\text{C}$$

Example III

- Topping a tank

An extension of the previous example has some fluid originally in the tank.

$$m_2(u+e_p+e_k)_2 – m_1(u+e_p+e_k)_1 + m_{out}(h+e_p+e_k)_{out} – m_{in}(h+e_p+e_k)_{in} = Q – (W_{sh} + W_{pv,system})$$

$$m_2u_2 – m_1u_1 – (m_2 – m_1)\, h_{in} = 0$$

Since m_2 and u_2 are unknown, one may have to use trial and error to solve.

Example IV

- A large unused exhibition hall (50 m x 40 m x 10 m) is to be prepared for a show and has to be heated from 0^oC to 25^oC. How many 1.5 kW portable heaters operating for 24 hrs would be needed for this job?
- Assume the pressure stays at 1 bar, air leaks out of the hall. Only account for the heating of air not walls, fixtures and furniture.

$$n_2 u_2 - n_1 u_1 + \int^{n_1 - n_2} h_{out}\, dn = Q$$

$$n_2 c_v T_2 - n_1 c_v T_1 + \int^{n_1 - n_2} c_p T dn = Q$$

changes as n changes

$$pv = nRT, \quad \text{or} \quad nT = \frac{pv}{R} = \text{const.} = n_1 T_1$$

$$dn_{\substack{\text{in}\\ \text{vessel}}} = \frac{n_1 T_1}{- T^2} dT = -dn_{out}$$

$$n_2 c_v T_2 \left(\frac{n_1 T_1}{n_2 T} \right) - n_1 c_v T_1 + c_p \int_{T_2}^{T_1} T \left(\frac{n_1 T_1}{- T^2} \right) dT = Q$$

$$Q = \frac{c_p pV}{R} = \frac{29.1(10000 \quad 0)(20000)}{8.314} \ln \frac{298}{273}$$

$$= 613 \times 10^6 \text{ J/day.}$$

$$\boxed{\frac{613 \times 10^6 \text{ J/day}}{24 \times 3600 \text{ s/day}} \left(\frac{\text{heater}}{1500\text{W}} \right) \approx 5}$$