Cascade and Minor Loop

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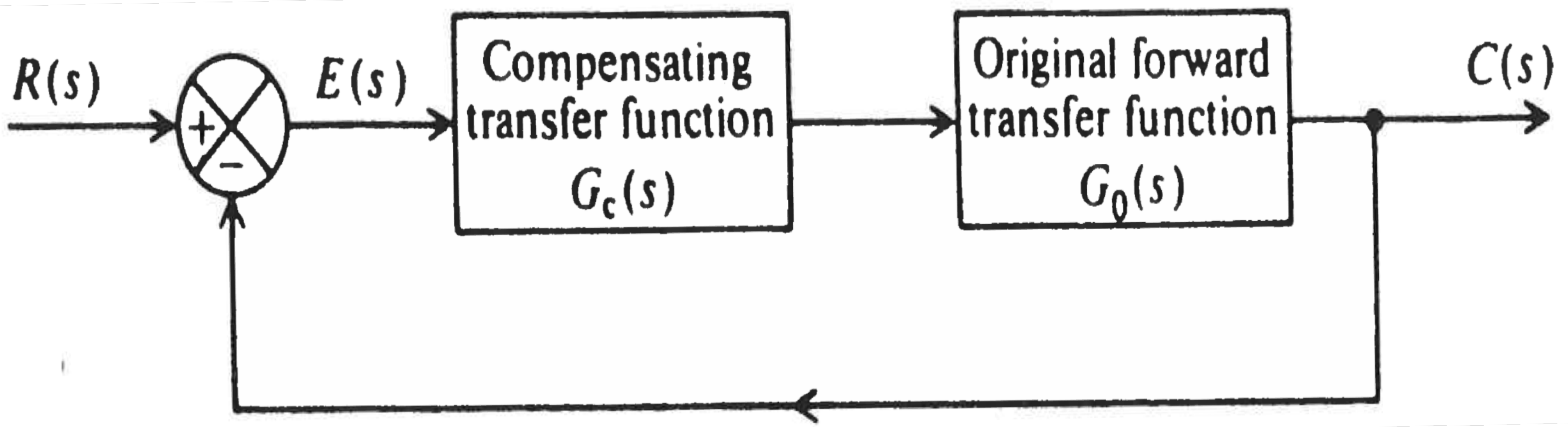
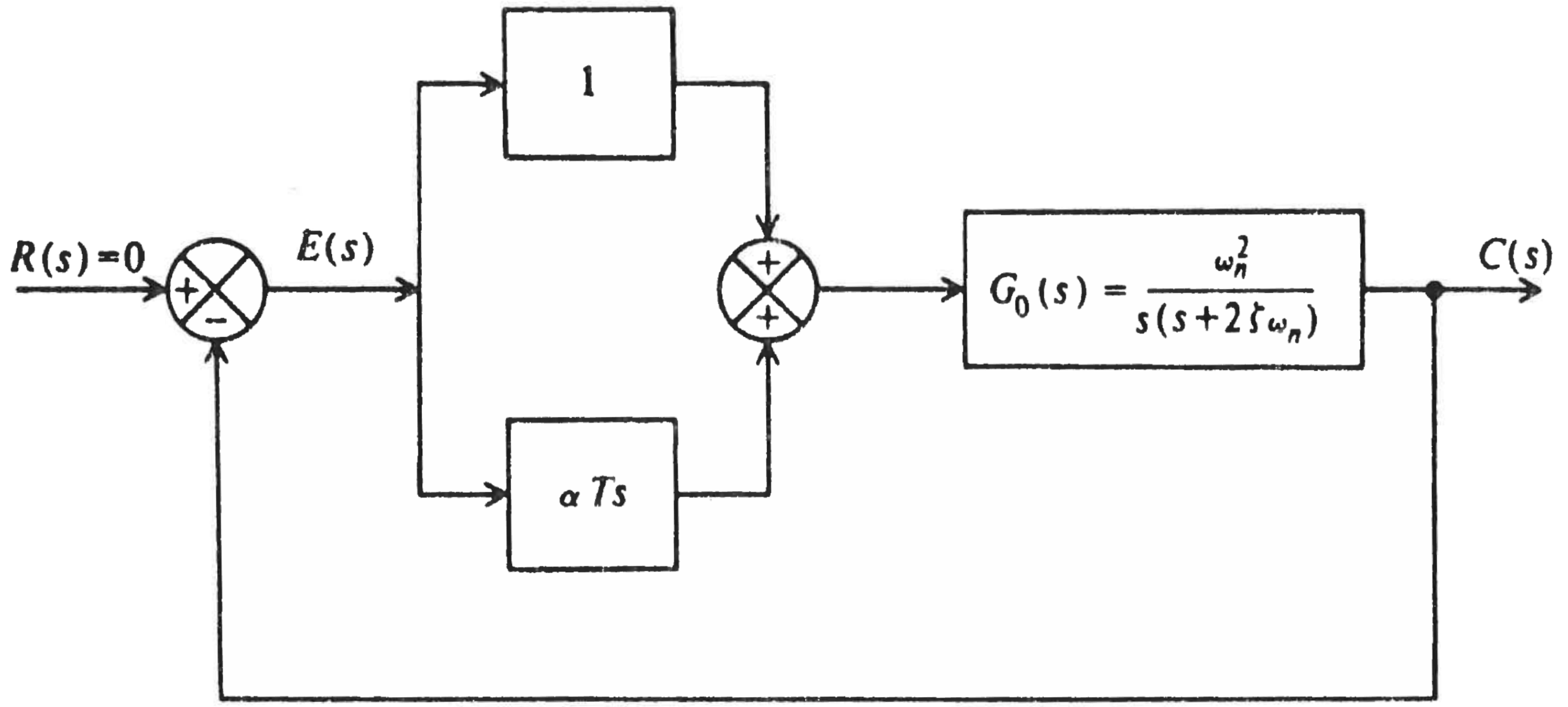




Propotional plus derivative control

$$\frac{C(s)}{R(s)}=\frac{G(s)}{1+G(s)}=\frac{\dfrac{\omega_n^2}{s(s+2\zeta\omega_n)}}{1+\dfrac{\omega_n^2}{s(s+2\zeta\omega_n)}}$$

$$=\frac{\omega_n^2}{s(s+2\zeta\omega_n)+\omega_n^2}=\frac{\omega_n^2}{s^2+(2\zeta\omega_n)s+\omega_n^2}$$

$$\frac{C(s)}{R(s)}=\frac{G(s)G_c(s)}{1+G(s)G_c(s)}\approx\frac{\dfrac{\omega_n^2(1+\alpha Ts)}{s(s+2\zeta\omega_n)}}{1+\dfrac{\omega_n^2(1+\alpha Ts)}{s(s+2\zeta\omega_n)}}$$

$$=\frac{\omega_n^2(1+\alpha Ts)}{s(s+2\zeta\omega_n)+\omega_n^2(1+\alpha Ts)}=\frac{\omega_n^2(1+\alpha Ts)}{s^2+(2\zeta\omega_n+\alpha T\omega_n^2)s+\omega_n^2}$$

$$\left(2\zeta\omega_n+\alpha T\omega_n^2\right)=2\zeta_{eq}\omega_n$$

$$\zeta_{eq}=\left(2\zeta+\alpha T\omega_n\right)/2=\zeta+\alpha T\omega_n/2$$

$$\frac{E(s)}{R(s)} = \frac{1}{1+G(s)} = \frac{1}{1+\dfrac{\omega_n^2}{s(s+2\zeta\omega_n)}}$$

$$= \frac{s(s+2\zeta\omega_n)}{s(s+2\zeta\omega_n)+\omega_n^2} = \frac{s(s+2\zeta\omega_n)}{s^2+(2\zeta\omega_n)s+\omega_n^2}$$

$$\boxed{R(s) = 1/s^2}$$

$$E(s) = \frac{(s+2\zeta\omega_n)}{s(s^2+(2\zeta\omega_n)s+\omega_n^2)}$$

$$e_{ss(ramp\ input)} = \lim_{s\to 0} sE(s)$$

$$= \lim_{s\to 0} s\frac{(s+2\zeta\omega_n)}{s(s^2+(2\zeta\omega_n)s+\omega_n^2)}$$

$$= \frac{2\zeta\omega_n}{\omega_n^2} = \frac{2\zeta}{\omega_n}$$

$$\frac{E(s)}{R(s)} = \frac{1}{1+G(s)G_c(s)} = \frac{1}{1+\dfrac{\omega_n^2(1+\alpha Ts)}{s(s+2\zeta\omega_n)(1+Ts)}}$$

$$= \frac{s(s+2\zeta\omega_n)(1+Ts)}{s(s+2\zeta\omega_n)(1+Ts)+\omega_n^2(1+\alpha Ts)}$$

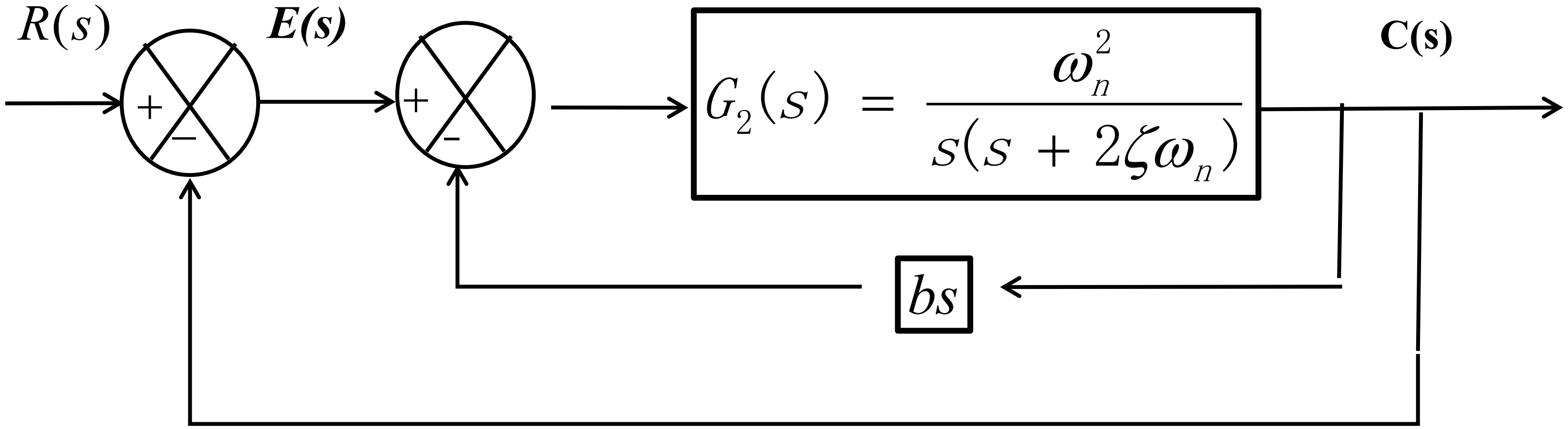
$$E(s) = \frac{(s+2\zeta\omega_n)(1+Ts)}{s(s(s+2\zeta\omega_n)(1+Ts)+\omega_n^2(1+\alpha Ts))}$$

$$e_{ss(ramp\ input)} = \lim_{s\to 0} sE(s)$$

$$= \lim_{s\to 0} s\frac{(s+2\zeta\omega_n)(1+Ts)}{s(s(s+2\zeta\omega_n)(1+Ts)+\omega_n^2(1+\alpha Ts))}$$

$$= \frac{2\zeta\omega_n}{\omega_n^2} = \frac{2\zeta}{\omega_n}$$

Minor- loop Feedback Compensation



$$G(s) = \frac{\omega_n^2}{s\left(s + 2\varsigma\omega_n\right)}$$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + \left(2\xi\omega_n + \omega_n^2 bs\right) + \omega_n^2}$$

$$2\zeta\omega_n + \omega_n^2 b = 2\zeta_{eq}\omega_n$$

$$\zeta_{eq} = \zeta + \frac{\omega_n b}{2}$$

$R(s) = 1/s^2$, **unit ramp input**

$$E(s) = \frac{1}{s}\left(\frac{s + 2\zeta\omega_n + b\omega_n^2}{s\left(s + 2\zeta\omega_n + b\omega_n^2\right) + \omega_n^2}\right)$$

$$e_{ss(ramp\ input)} = \lim_{s\to 0} sE(s) = \frac{2\zeta\omega_n + b\omega_n^2}{\omega_n^2} = \frac{2\zeta}{\omega_n} + b$$