

# **Ideal Gas Mixture and Psychrometric Applications**

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# **IDEAL GAS MIXTURE**

# Relating $p$ , $V$ , and $T$ for Ideal Gas Mixtures

- **The Dalton model:** the Dalton model assumes that each mixture component behaves as an ideal gas as if it were alone at the temperature  $T$  and volume  $V$  of the mixture. Individual components would not exert the mixture pressure  $p$  but rather a partial pressure.

$$p_i = \frac{n_i RT}{V}$$

$$\frac{p_i}{p} = \frac{n_i RT / V}{nRT / V} = \frac{n_i}{n} = y_i$$

$$p_i = y_i p$$

# Ideal Gas Mixture Properties

- Consider the internal energy and enthalpy of an ideal gas mixture. Recall that

$$dE = TdS - pdV + \sum_{i=1}^r \mu_i dn_i$$

$$dH = TdS + Vdp + \sum_{i=1}^r \mu_i dn_i$$

- For ideal gas,  $pv=nRT$ , The components of the mixture exist at the same temperature as the mixture.

- Therefore,

$$u_m(T_m) = \sum_i \omega_i u_i(T_m) \quad \text{or} \quad \bar{u}_m(T_m) = \sum_i y_i \bar{u}_i(T_m)$$

$$h_m(T_m) = \sum_i \omega_i h_i(T_m) \quad \text{or} \quad \bar{h}_m(T_m) = \sum_i y_i \bar{h}_i(T_m)$$

- What about the entropy of an ideal gas?

$$\bar{s}_m(T_m, p_m) = \sum_i y_i \bar{s}_i(T_m, ?)$$

- The mixture must obey **the Dalton model**

$$n_m = \frac{p_m V_m}{RT_m}, \quad n_i = \frac{p_i V_m}{RT_m}$$

- Therefore,

$$S_m(T_m, p_m) = \sum_i \omega_i S_i(T_m, p_i) \quad \text{or} \quad \bar{S}_m(T_m, p_m) = \sum_i y_i \bar{S}_i(T_m, p_i)$$

- Then

$$\bar{S}_i^{ig}(T, p) - S_i^{ig}(T, p) = -R \ln \frac{p}{p_i} = R \ln y_i$$

$$S_i^{ig}(T, p_i) - S_i^{ig}(T, p) = -R \ln \frac{p}{p_i} = R \ln y_i$$

} Obey the 2<sup>nd</sup> law

- where

$$dH = -T dS + V dp + \sum_{i=1}^r \mu_i dn_i$$

$$dS \Big|_{T, n_i} = \frac{V}{T} dp = \frac{R}{p} dp = R \ln p$$

- For the Gibbs energy of an ideal-gas mixture
- $G^{ig} = H^{ig} - TS^{ig}$ ,

$$\bar{G}_i^{ig} = \bar{H}_i^{ig} - T\bar{S}_i^{ig}$$

$$\bar{G}_i^{ig} = H_i^{ig} - TS_i^{ig} + RT \ln y_i$$

$$\mu_i^{ig} \equiv \bar{G}_i^{ig} = G_i^{ig} + RT \ln y_i$$

# summary

Using Dalton's Law ...

$$u_m(T_m) = \sum_k w_k u_k(T_k) \quad \text{or} \quad \bar{u}_m = \sum_k y_k \bar{u}_k(T_k)$$

$$h_m(T_m) = \sum_k w_k h_k(T_k) \quad \text{or} \quad \bar{h}_m = \sum_k y_k \bar{h}_k(T_k)$$

$$c_{pm}(T_m) = \sum_k w_k c_{pk}(T_k) \quad \text{or} \quad \bar{c}_{pm} = \sum_k y_k \bar{c}_{pk}(T_k)$$

$$c_{vm}(T_m) = \sum_k w_k c_{vk}(T_k) \quad \text{or} \quad \bar{c}_{vm} = \sum_k y_k \bar{c}_{vk}(T_k)$$

$$s_m(T_m, P_m) = \sum_k w_k s_k(T_m, P_k) \quad \text{or} \quad \bar{s}_m(T_m, P_m) = \sum_k y_k \bar{s}_k(T_m, P_k)$$

- **Amagat model** is that each mixture component behaves as an ideal gas as if it existed separately at the pressure  $p$  and temperature  $T$  of the mixture.

$$V_i = \frac{n_i RT}{p}$$

$$\frac{V_i}{V} = \frac{n_i RT / p}{n RT / p} = \frac{n_i}{n} = y_i$$

$$V = \sum V_i$$

# EXAMPLE 1

- Converting Mole Fractions to Mass Fractions
- The molar analysis of the gaseous products of combustion of a certain hydrocarbon fuel is CO<sub>2</sub>, 0.08; H<sub>2</sub>O, 0.11; O<sub>2</sub>, 0.07; N<sub>2</sub>, 0.74.
  - (a) Determine the apparent molecular weight of the mixture.
  - (b) Determine the composition in terms of mass fractions.
- Solution
  - (a)  $M = 0.08(44) + 0.11(18) + 0.07(32) + 0.74(28)$   
 $= 28.46 \text{ g/mol}$

– (b)

component	$n_i$	×	$M_i$	=	$m_i$	$\omega_i(\%)$
CO <sub>2</sub>	0.08	×	44	=	3.52	12.37
H <sub>2</sub> O	0.11	×	18	=	1.98	6.96
O <sub>2</sub>	0.07	×	32	=	2.24	7.87
N <sub>2</sub>	0.74	×	28	=	20.72	72.8
	1.00				28.46	100

- Converting Mass Fractions to Mole Fractions
- A gas mixture has the following composition in terms of mass fractions: H<sub>2</sub>, 0.10; N<sub>2</sub>, 0.60; CO<sub>2</sub>, 0.30. Determine
  - (a) the composition in terms of mole fractions and
  - (b) the apparent molecular weight of the mixture.

- Solution

Component	$m_i$	$\div$	$M_i$	=	$n_i$	$y_i(\%)$
H <sub>2</sub>	10	$\div$	2	=	5.00	63.9
N <sub>2</sub>	60	$\div$	28	=	2.14	27.4
CO <sub>2</sub>	30	$\div$	44	=	0.68	8.7
	100				7.82	100

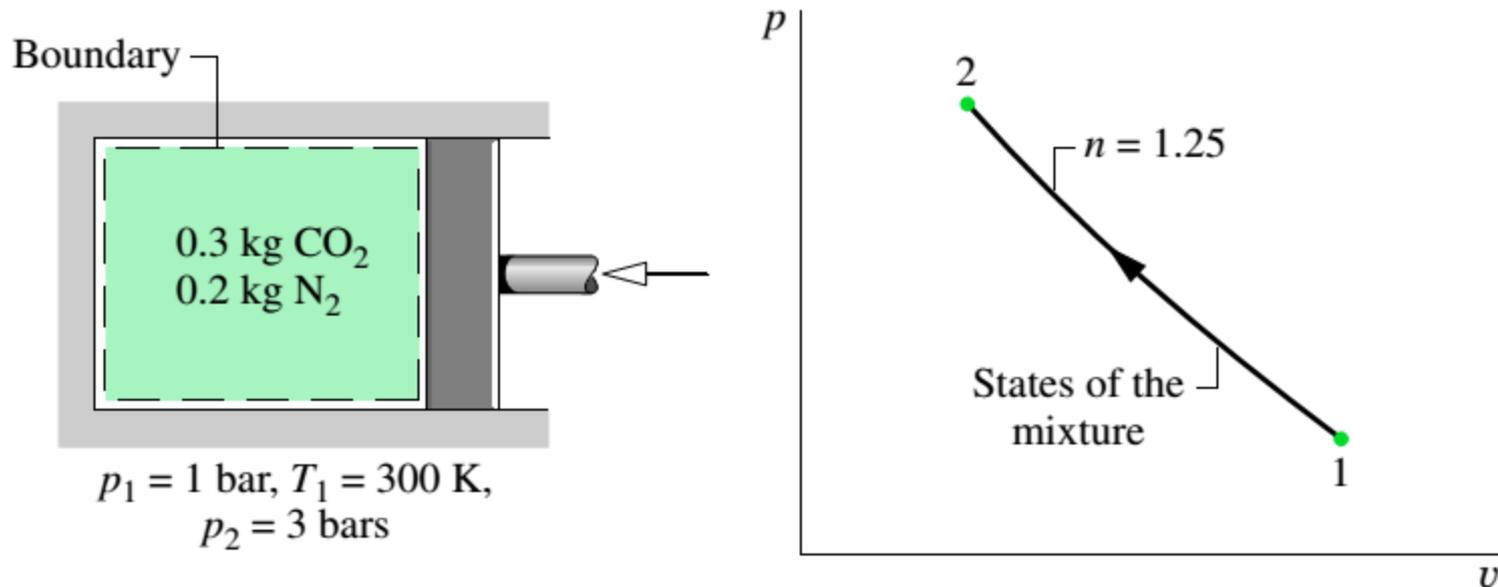
– (b)  $M = m/n = 100/78.2 = 12.79 \text{ g/mol}$

# Quiz XII

- 1) A mixture of gases has the following analysis on mass basis.
- Component       $\text{CO}_2$     $\text{H}_2\text{O}$     $\text{O}_2$        $\text{N}_2$        $\text{H}_2$
- Mass %              20      10      15      35      20
- Calculate the composition in terms of mole fractions and the molar mass of the mixture.

# EXAMPLE 2

- Compressing an Ideal Gas Mixture
- A mixture of 0.3 kg of carbon dioxide and 0.2 kg of nitrogen is compressed from  $p_1=1$  bar,  $T_1=300$  K to  $p_2=3$  bars in a **polytropic** process for which  $k=1.25$ . Determine
  - (a) the final temperature, in K,
  - (b) the work, in kJ,
  - (c) the heat transfer, in kJ,
  - (d) the change in entropy of the mixture, in kJ/K.



- Assume
- As shown in the accompanying figure, the system is the mixture of CO<sub>2</sub> and N<sub>2</sub>. The mixture composition remains constant during the compression.
- Each mixture component behaves as if it were an ideal gas occupying the entire system volume at the mixture temperature. The overall mixture acts as an ideal gas.
- The compression process is a polytropic process for which  $k=1.25$ .
- The changes in kinetic and potential energy between the initial and final states can be ignored.

- (a) For an ideal gas, the temperatures and pressures at the end states of a polytropic process,

$$T_2 = T_1 \left( \frac{p_2}{p_1} \right)^{\left( \frac{k-1}{k} \right)} = 300 \left( \frac{3}{1} \right)^{0.2} = 374K$$

- (b)  $W = \int p dV$

$$pV^n = \text{const}$$

$$W = \frac{p_2 V_2 - p_1 V_1}{1-k} = \frac{\frac{m}{M} R (T_2 - T_1)}{1-k}$$

- $m = 0.3 + 0.2$ ,  $n = n_{\text{CO}_2} + n_{\text{N}_2}$ ,  $M = m/n$

$$W = \frac{\frac{(0.5 \text{ kg})}{35.79 \text{ kg/mol}} (8.314 \text{ kJ/mol/K}) (374 \text{ K} - 300 \text{ K})}{1-1.25} = -34.21 \text{ kJ}$$

- (c) The change in internal energy of the mixture equals the sum of the internal energy changes of the components.

$$\Delta U = n_{CO_2} [\bar{u}_{CO_2}(T_2) - \bar{u}_{CO_2}(T_1)] + n_{N_2} [\bar{u}_{N_2}(T_2) - \bar{u}_{N_2}(T_1)]$$

$$\Delta U = (0.0068)(9198 - 6939) + (0.0071)(7770 - 6229)$$

$$= 26.3 \text{ kJ}$$

- Inserting values for  $\Delta U$  and  $W$  into the expression for  $Q$ :

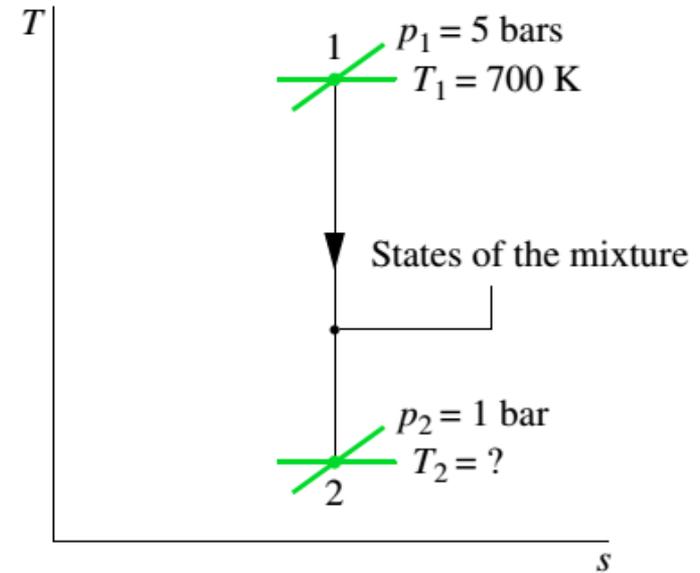
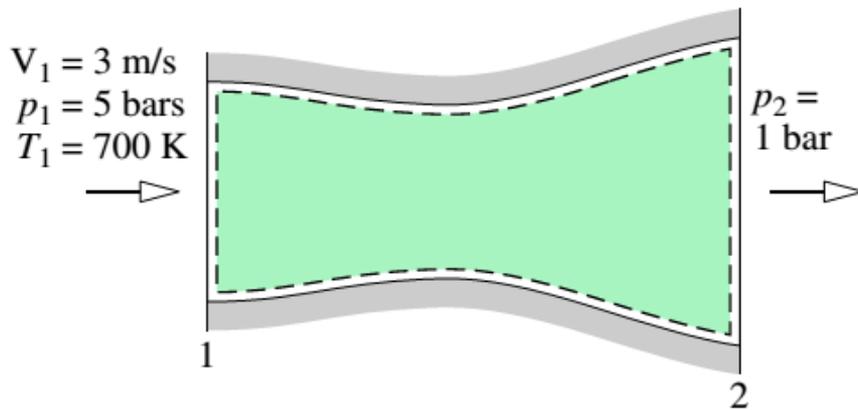
$$Q = 26.3 - 34.21 = 7.91 \text{ kJ}$$

- (d)
- $$\Delta S = n_{CO_2} \Delta \bar{s}_{CO_2} + n_{N_2} \Delta \bar{s}_{N_2}$$
- $$= 0.0068(222.475 - 213.915 - 8.314 \frac{3}{1})$$
- $$+ 0.0071(198.105 - 191.682 - 8.314 \frac{3}{1})$$
- $$= -0.0231 \text{ kJ} / K$$

**Entropy decreases** in the process because entropy is transferred from the system accompanying heat transfer.

# EXAMPLE 3

- Gas Mixture Expanding Isentropically through a Nozzle
- A gas mixture consisting of  $\text{CO}_2$  and  $\text{O}_2$  with mole fractions 0.8 and 0.2, respectively, expands isentropically and at steady state through a nozzle from 700 K, 5 bars, 3 m/s to an exit pressure of 1 bar. Determine
  - (a) the temperature at the nozzle exit, in K,
  - (b) the entropy changes of the  $\text{CO}_2$  and  $\text{O}_2$  from inlet to exit, in
  - (c) the exit velocity, in m/s.



- Assume
- The control volume shown by the dashed line on the accompanying figure operates at steady state.
- The mixture composition remains constant as the mixture expands isentropically through the nozzle. The overall mixture and each mixture component act as ideal gases. The state of each component is defined by the temperature and the partial pressure of the component.
- The change in potential energy between inlet and exit can be ignored.

- solution
- (a) The temperature at the exit can be determined using the fact that the expansion occurs isentropically

$$\bar{s}_2 - \bar{s}_1 = y_{O_2} \Delta \bar{s}_{O_2} + y_{CO_2} \Delta \bar{s}_{CO_2} = 0$$

therefore

$$\begin{aligned}
 & y_{O_2} \left[ \bar{s}_{O_2}^0(T_2) - \bar{s}_{O_2}^0(T_1) - R \ln \frac{p_2}{p_1} \right] \\
 & \quad + y_{CO_2} \left[ \bar{s}_{CO_2}^0(T_2) - \bar{s}_{CO_2}^0(T_1) - R \ln \frac{p_2}{p_1} \right] = 0 \\
 & y_{O_2} \bar{s}_{O_2}^0(T_2) + y_{CO_2} \bar{s}_{CO_2}^0(T_2) = y_{O_2} \bar{s}_{O_2}^0(T_1) + y_{CO_2} \bar{s}_{CO_2}^0(T_1) \\
 & \quad + (y_{O_2} + y_{CO_2}) R \ln \frac{p_2}{p_1}
 \end{aligned}$$

$$T_1 = 700^\circ\text{C}$$

$$\begin{aligned} & (0.2)\bar{s}_{O_2}^0(T_2) + (0.8)\bar{s}_{CO_2}^0(T_2) \\ &= (0.2)(231.358) + (0.8)(250.663) + (0.2 + 0.8)8.314 \ln \frac{1}{5} \end{aligned}$$

Through iteration,  $T_2 = 517.6 \text{ K}$

•(b) The change in the specific entropy for each of the components can be determined using

$$\Delta\bar{s}_{O_2} = \bar{s}_{O_2}^0(T_2) - \bar{s}_{O_2}^0(T_1) - R \ln \frac{p_2}{p_1}$$

$$\Delta\bar{s}_{O_2} = 221.667 - 231.358 - 8.314 \ln(0.2) = 3.69 \text{ kJ / kmol} \cdot \text{K}$$

$$\Delta\bar{s}_{CO_2} = \bar{s}_{CO_2}^0(T_2) - \bar{s}_{CO_2}^0(T_1) - R \ln \frac{p_2}{p_1}$$

$$\Delta\bar{s}_{CO_2} = 236.365 - 250.663 - 8.314 \ln(0.2) = -0.92 \text{ kJ / kmol} \cdot \text{K}$$

- (c) The energy rate balance for the one-inlet, one-exit control volume at steady state

$$0 = h_1 - h_2 + \frac{v_1^2 - v_2^2}{2}$$

$$h_1 - h_2 = \frac{\bar{h}_1 - \bar{h}_2}{M} = \frac{1}{M} [y_{O_2} (\bar{h}_1 - \bar{h}_2)_{O_2} + y_{CO_2} (\bar{h}_1 - \bar{h}_2)_{CO_2}]$$

- where apparent molecular weight  
 $M = (0.8)44 + 0.2(32) = 41.6 \text{ kg/kmol}$

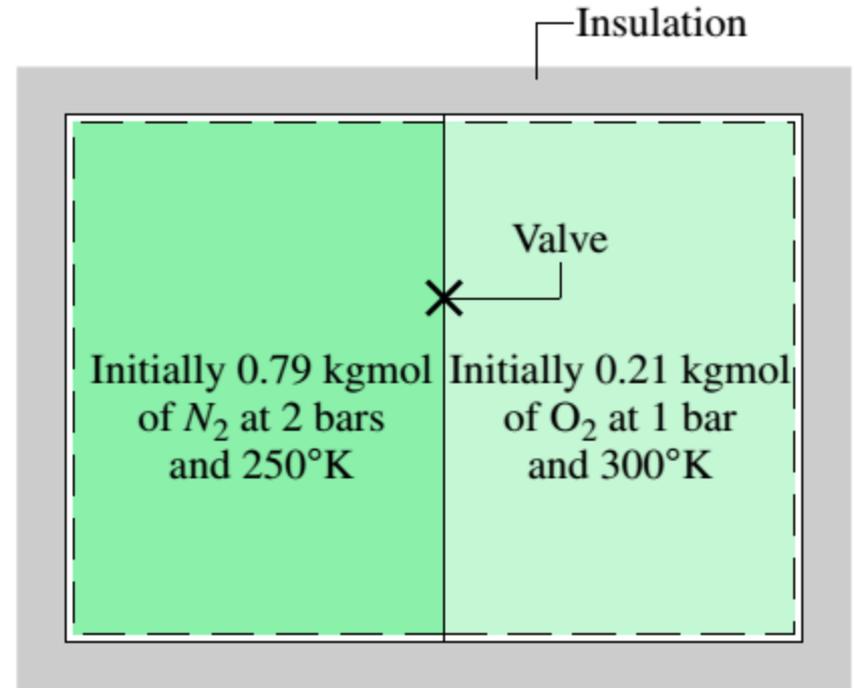
$$\begin{aligned} h_1 - h_2 &= \frac{1}{41.6} [0.2(21184 - 15320) + 0.8(27125 - 18468)] \\ &= 194.7 \text{ kJ/kg} \end{aligned}$$

- $v_2 = 624 \text{ m/s}$

# EXAMPLE 4

- Adiabatic Mixing at Constant Total Volume
- Two rigid, insulated tanks are interconnected by a valve. Initially 0.79 kmol of  $N_2$  at 2 bars and 250 K fills one tank. The other tank contains 0.21 kmol of  $O_2$  at 1 bar and 300 K. The valve is opened and the gases are allowed to mix until a final equilibrium state is attained. During this process, there are no heat or work interactions between the tank contents and the surroundings. Determine
  - (a) the final temperature of the mixture, in K,
  - (b) the final pressure of the mixture, in atm,
  - (c) the amount of entropy produced in the mixing process, in kJ/K.

- Assumptions:
- 1. The system is taken to be the nitrogen and the oxygen together.
- 2. When separate, each of the gases behaves as an ideal gas. The final mixture also acts as an ideal gas. Each mixture component occupies the total volume and exhibits the mixture temperature.
- 3. No heat or work interactions occur with the surroundings, and there are no changes in kinetic and potential energy.



- Solution
- (a) The final temperature of the mixture can be determined from an energy balance. With assumption 3, the closed system energy balance reduces to

$$\Delta U = U_2 - U_1 = \cancel{Q} - \cancel{W} = 0$$

$$U_1 = n_{N_2} \bar{u}_{N_2}(T_{1,N_2}) + n_{O_2} \bar{u}_{O_2}(T_{1,O_2})$$

$$U_2 = n_{N_2} \bar{u}_{N_2}(T_2) + n_{O_2} \bar{u}_{O_2}(T_2)$$

$$n_{N_2} [\bar{u}_{N_2}(T_2) - \bar{u}_{N_2}(T_{1,N_2})] + n_{O_2} [\bar{u}_{O_2}(T_2) - \bar{u}_{O_2}(T_{1,O_2})] = 0$$

- or

$$n_{N_2} \bar{c}_{v,N_2}(T_2 - T_{1,N_2}) + n_{O_2} \bar{c}_{v,O_2}(T_2 - T_{1,O_2}) = 0$$

$$n_{N_2} \bar{c}_{v,N_2} (T_2 - T_{1,N_2}) + n_{O_2} \bar{c}_{v,O_2} (T_2 - T_{1,O_2}) = 0$$

$$T_2 = \frac{n_{N_2} \bar{c}_{v,N_2} T_{1,N_2} + n_{O_2} \bar{c}_{v,O_2} T_{1,O_2}}{n_{N_2} \bar{c}_{v,N_2} + n_{O_2} \bar{c}_{v,O_2}}$$

$$T_2 = \frac{(0.79 \text{ kmol}) \left( \frac{20.82 \text{ kJ}}{\text{kmol} \cdot \text{K}} \right) (250 \text{ K}) + (0.21 \text{ kmol}) \left( \frac{20.99 \text{ kJ}}{\text{kmol} \cdot \text{K}} \right) (300 \text{ K})}{(0.79 \text{ kmol}) \left( \frac{20.82 \text{ kJ}}{\text{kmol} \cdot \text{K}} \right) + (0.21 \text{ kmol}) \left( \frac{20.99 \text{ kJ}}{\text{kmol} \cdot \text{K}} \right)}$$

$$= 261 \text{ K}$$

• (b)

$$V = \frac{n_{N_2} RT_{1,N_2}}{p_{N_2}} + \frac{n_{O_2} RT_{1,O_2}}{p_{O_2}}$$

$$p_2 = \frac{nRT_2}{V}$$

$$p_2 = \frac{(n_{N_2} + n_{O_2})T_2}{\left( \frac{n_{N_2} T_{1,N_2}}{p_{N_2}} + \frac{n_{O_2} T_{1,O_2}}{p_{O_2}} \right)}$$

$$= \frac{(1.0 \text{ kmol})(261 \text{ K})}{\left( \frac{(0.79 \text{ kmol})(250 \text{ K})}{2 \text{ bars}} + \frac{(0.21 \text{ kmol})(300 \text{ K})}{1 \text{ bars}} \right)}$$

$$= 1.62 \text{ bars}$$

- (c) the closed system form of the entropy balance

$$S_2 - S_1 = \int_1^2 \left( \frac{\delta Q}{T} \right) + \sigma$$

0, adiabatic

- The initial entropy of the system,  $S_1$ , is the sum of the entropies of the gases at the respective initial states

$$S_1 = n_{N_2} \bar{s}_{N_2}(T_{1,N_2}, p_{N_2}) + n_{O_2} \bar{s}_{O_2}(T_{1,O_2}, p_{O_2})$$

- The final entropy of the system,  $S_2$ , is the sum of the entropies of the individual components, each evaluated at the final mixture temperature and the partial pressure of the component in the mixture

$$S_2 = n_{N_2} \bar{s}_{N_2}(T_2, y_{N_2} p_2) + n_{O_2} \bar{s}_{O_2}(T_2, y_{O_2} p_2)$$

- entropy produced

$$\sigma = n_{N_2} \left( \bar{s}_{N_2}(T_2, y_{N_2} p_2) - \bar{s}_{N_2}(T_{1,N_2}, p_{N_2}) \right) + n_{O_2} \left( \bar{s}_{O_2}(T_2, y_{O_2} p_2) - \bar{s}_{O_2}(T_{1,O_2}, p_{O_2}) \right)$$

- or

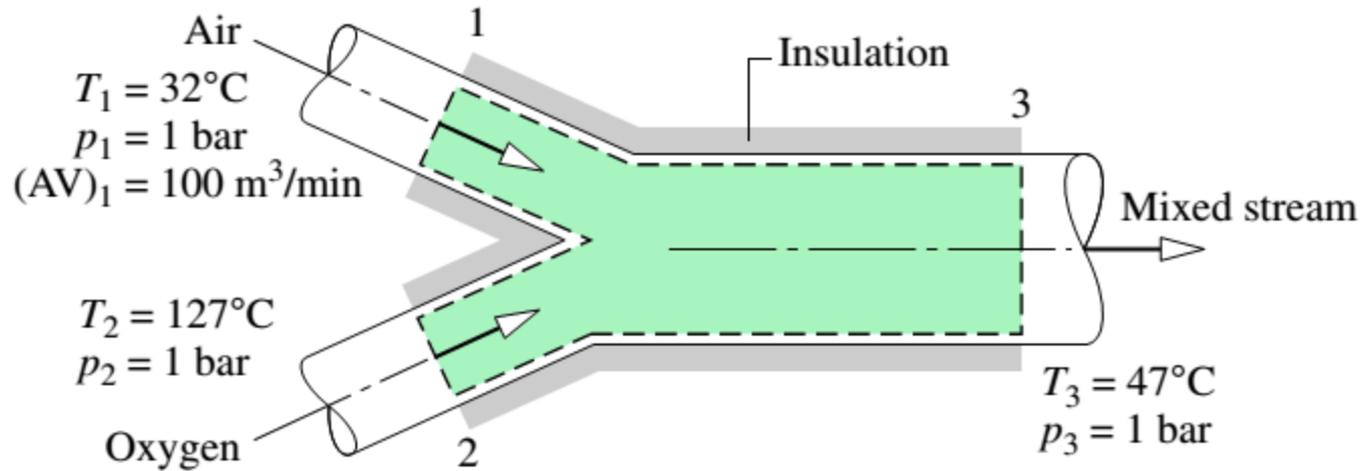
$$\sigma = n_{N_2} \left( \bar{c}_{p,N_2} \ln \frac{T_2}{T_{1,N_2}} - R \ln \frac{y_{N_2} p_2}{p_{N_2}} \right) + n_{O_2} \left( \bar{c}_{p,O_2} \ln \frac{T_2}{T_{1,O_2}} - R \ln \frac{y_{O_2} p_2}{p_{O_2}} \right)$$

constant p                      constant T

$$\begin{aligned}
\sigma &= n_{N_2} \left( \bar{c}_{p,N_2} \ln \frac{T_2}{T_{1,N_2}} - R \ln \frac{y_{N_2} p_2}{p_{N_2}} \right) \\
&+ n_{O_2} \left( \bar{c}_{p,O_2} \ln \frac{T_2}{T_{1,O_2}} - R \ln \frac{y_{O_2} p_2}{p_{O_2}} \right) \\
\sigma &= (0.79) \left( (29.13) \ln \frac{261}{250} - 8.314 \ln \frac{(0.79)(1.62)}{2} \right) \\
&+ (0.21) \left( (29.30) \ln \frac{261}{300} - 8.314 \ln \frac{(0.21)(1.62)}{1} \right) \\
&= 5.0 \text{ kJ} / \text{K}
\end{aligned}$$

# EXAMPLE 5

- Adiabatic Mixing of Two Streams
- At steady state,  $100 \text{ m}^3/\text{min}$  of dry air at  $32^\circ\text{C}$  and 1 bar is mixed adiabatically with a stream of oxygen ( $\text{O}_2$ ) at  $127^\circ\text{C}$  and 1 bar to form a mixed stream at  $47^\circ\text{C}$  and 1 bar. Kinetic and potential energy effects can be ignored. Determine
  - (a) the mass flow rates of the dry air and oxygen, in  $\text{kg}/\text{min}$ ,
  - (b) the mole fractions of the dry air and oxygen in the exiting mixture, and
  - (c) the time rate of entropy production, in  $\text{kJ}/\text{K min}$



Assume

- 1. steady state.
- 2.No heat transfer occurs with the surroundings.
- 3.Kinetic and potential energy effects can be ignored,
- 4.The entering gases can be regarded as ideal gases. The exiting mixture can be regarded as an ideal gas mixture.
- 5.The dry air is treated as a pure component.

- Solution
- the specific volume of the air at 1 is

$$v_{air,1} = \frac{1}{M_{air}} \frac{RT_1}{p_1} = \frac{\left( \frac{8314 \text{ N} \cdot \text{m}}{28.97 \text{ kg} \cdot \text{K}} \right) (305 \text{ K})}{10^5 \text{ N} / \text{m}^2} = 0.875 \frac{\text{m}^3}{\text{kg}}$$

- The mass flow rate of the dry air entering is

$$\dot{m}_{air,1} = \frac{(AV)_1}{v_{air,1}} = \frac{100 \text{ m}^3 / \text{min}}{0.875 \text{ m}^3 / \text{kg}} = 114.29 \frac{\text{kg}}{\text{min}}$$

- From mass balances,

$$\dot{m}_{air,1} = \dot{m}_{air,3}$$

$$\dot{m}_{O_2,2} = \dot{m}_{O_2,3}$$

- The enthalpy of the mixture at the exit is evaluated by summing the contributions of the air and oxygen, each at the mixture temperature.

$$\dot{m}_{air,1}h_a(T_1) + \dot{m}_{O_2,2}h_O(T_2) = (\dot{m}_{air,3}h_a(T_3) + \dot{m}_{O_2,3}h_O(T_3))$$

$$\dot{m}_{O_2,2=3} = \dot{m}_{air,1=3} \left( \frac{h_a(T_3) - h_a(T_1)}{h_O(T_2) - h_O(T_3)} \right) = 23.1 \text{ kg / min}$$

- (b) the mole fractions of the dry air and oxygen in the exiting mixture

$$\dot{n}_a = \frac{\dot{m}_{air}}{M_{air}} = \frac{114.29}{28.97} = 3.95 \text{ kmol / min}$$

$$\dot{n}_{O_2} = \frac{\dot{m}_{O_2}}{M_{O_2}} = \frac{23.1}{32} = 0.72 \text{ kmol / min}$$

$$\dot{n} = \dot{n}_a + \dot{n}_{O_2} = 3.95 + 0.72 = 4.67 \text{ kmol / min}$$

$$y_{air} = \frac{\dot{n}_a}{\dot{n}} = 0.846 \quad \text{and} \quad y_{O_2} = \frac{\dot{n}_{O_2}}{\dot{n}} = 0.154$$

- (c) The specific entropy of each component in the exiting ideal gas mixture is evaluated at its partial pressure in the mixture and at the mixture temperature

$$\dot{m}_{air,1}s_a(T_1, p_1) + \dot{m}_{O_2,2}s_O(T_2, p_2) - (\dot{m}_{air,3}s_a(T_3, p_3) + \dot{m}_{O_2,3}s_O(T_3, p_3)) + \dot{\sigma} = 0$$

$$\dot{\sigma} = \dot{m}_{air,1=3} [s_a(T_3, p_3) - s_a(T_1, p_1)] + \dot{m}_{O_2,2=3} [s_O(T_3, p_3) - s_O(T_1, p_1)]$$

- Since  $p_1 = p_3$ , the specific entropy change of the dry air is

$$s_a(T_3, y_a p_3) - s_a(T_1, p_1) = s_a^0(T_3) - s_a^0(T_1) - \frac{R}{M_{air}} \ln \frac{y_a p_3}{p_1}$$

2<sup>nd</sup> law

- since  $p_2 = p_3$ , the specific entropy change of the oxygen is

$$s_O(T_3, y_O p_3) - s_O(T_2, p_2) = s_O^0(T_3) - s_O^0(T_2) - \frac{R}{M_{O_2}} \ln y_O$$

- The rate of entropy production becomes

$$\begin{aligned}\dot{\sigma} &= \dot{m}_{air} \left[ s_a^0(T_3) - s_a^0(T_1) - \frac{R}{M_{air}} \ln y_a \right] + \dot{m}_{O_2} \left[ s_O^0(T_3) - s_O^0(T_2) - \frac{R}{M_{O_2}} \ln y_O \right] \\ &= 17.42 \frac{kJ}{K \cdot \text{min}}\end{aligned}$$

# **PSYCHROMETRIC APPLICATIONS**

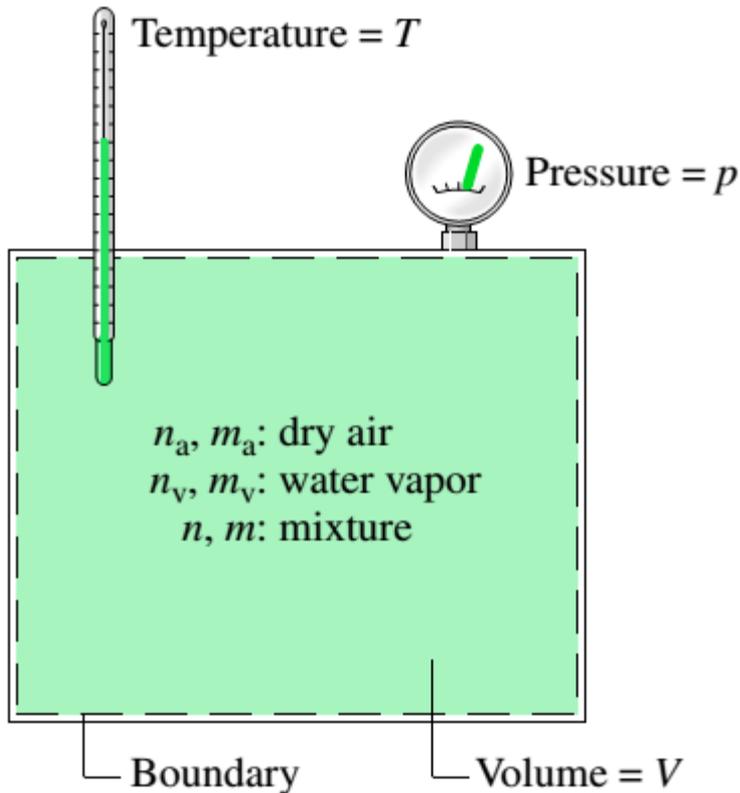
# Psychrometrics

- Psychrometrics is the determination of physical and thermodynamic properties of gas-vapor mixtures, study of systems involving mixtures of dry air and water vapor. A condensed water phase also may be present.
- Such systems is essential for the analysis and design of air-conditioning devices, cooling towers, and industrial processes requiring close control of the vapor content in air.

# Moist Air

- The term moist air refers to a mixture of dry air and water vapor in which the dry air is treated as if it were a pure component.
- As can be verified by reference to appropriate property data, the overall mixture and each mixture component behave as ideal gases at the states under present consideration.
- Accordingly, for the applications to be considered, the ideal gas mixture concepts introduced previously apply directly.

# Humidity Ratio, Relative Humidity, and Mixture Enthalpy



- Humidity ratio  $\omega$

$$\omega = \frac{m_{\text{vapor}}}{m_{\text{air}}} = \frac{M_v p_v V / RT}{M_a p_a V / RT}$$

$$= \frac{M_v p_v}{M_a p_a}$$

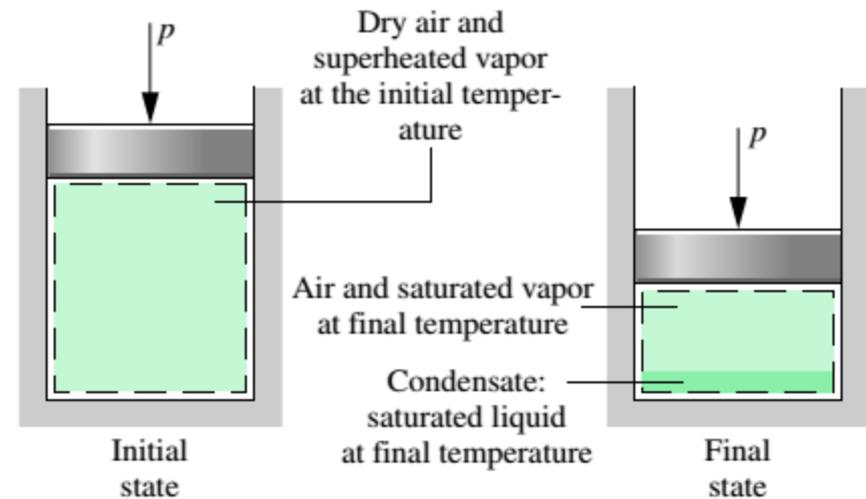
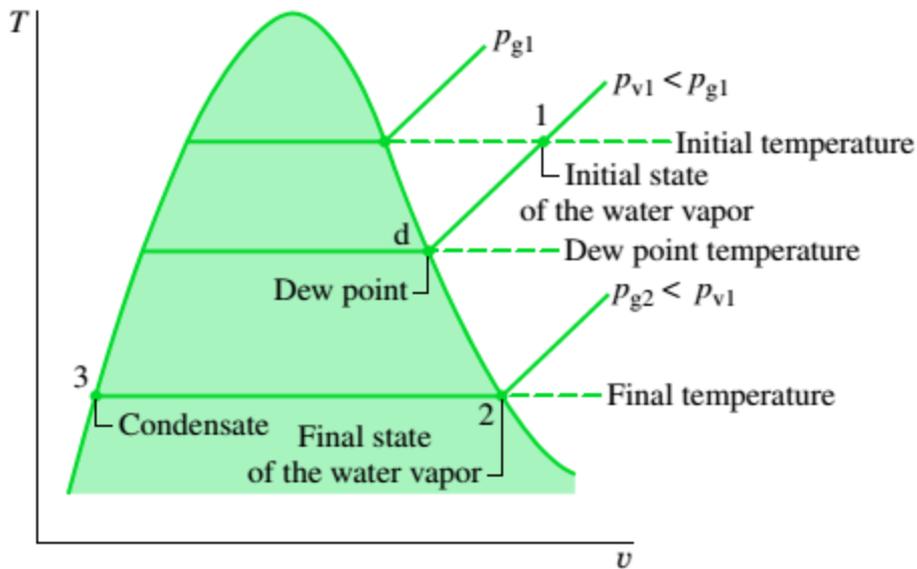
- Relative humidity

$$\phi = \frac{y_v}{y_{v,sat}} \Big|_{T,p} = \frac{p_v}{p_g} \Big|_{T,p}$$

- Mixture Enthalpy

$$H = H_a + H_v = m_a h_a + m_v h_v$$

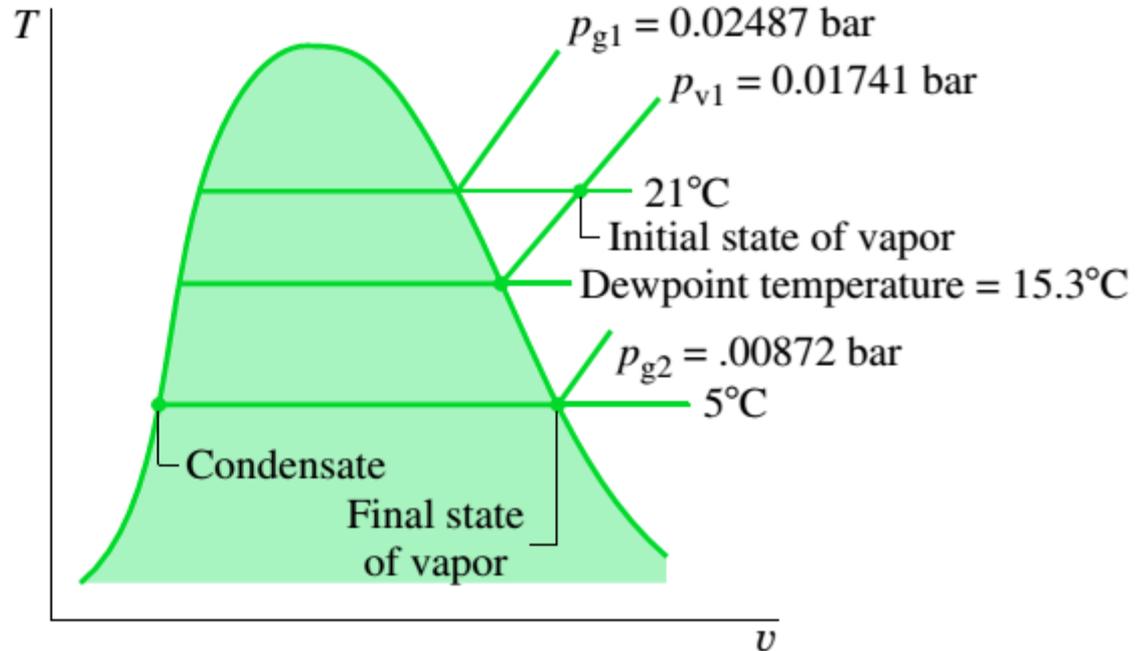
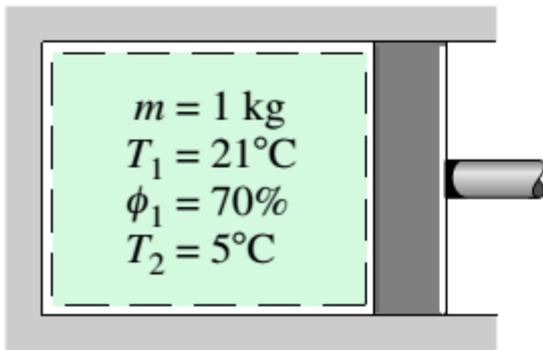
$$\frac{H}{m_a} = h_a + \frac{m_v}{m_a} h_v = h_a + \omega h_v$$



- partial condensation of the water vapor can occur when the temperature is reduced. Water vapor would cool at constant  $p_v$  from state 1 to state d, called the dew point.
- the system would be cooled below the dew point temperature, some of the water vapor would condense. The vapor that remains can be regarded as saturated at the final temperature, state 2.

# Example 1

- Cooling Moist Air at Constant Pressure
- A 1 kg sample of moist air initially at 21°C, 1 bar, and 70% relative humidity is cooled to 5°C while keeping the pressure constant. Determine
- (a) the initial humidity ratio,
- (b) the dew point temperature, in °C, and
- (c) the amount of water vapor that condenses, in kg.



- (a) The partial pressure of the water vapor,  $p_{v1}$  can be found from the given relative humidity and  $p_g$  from Table at  $21^\circ\text{C}$ , 1 bar.

$$p_{v1} = \phi p_g = (0.7)(0.02487 \text{ bar}) = 0.01741 \text{ bar}$$

$$\omega_1 = \frac{M_v p_v}{M_a p_a} = 0.622 \frac{p_{v1}}{p - p_{v1}} = 0.011$$

- (b) The dew point temperature is the saturation temperature corresponding to the partial pressure,  $p_{v1}$ . Interpolation in Table gives  $T=15.3^{\circ}\text{C}$ .
- (c) The amount of condensate,  $m_w$ , equals the difference between the initial amount of water vapor in the sample,  $m_{v1}$ , and the final amount of water vapor,  $m_{v2}$  ,

$$m_w = m_{v1} - m_{v2}$$

$$m_a + m_{v1} = 1 \text{ lb}$$

$$\omega_1 = m_{v1} / m_a$$

$$m_{v1} / \omega_1 + m_{v1} = m_{v1} (1 / \omega_1 + 1) = 1 \text{ kg}$$

$$m_{v1} = \frac{1 \text{ kg}}{1 / 0.011 + 1} = 0.0109 \text{ kg}$$

$$m_a = 1 - 0.0109 = 0.9891 \text{ kg}$$

- the partial pressure of the water vapor remaining in the system at the final state is the saturation pressure corresponding to 5°C:  $p_g=0.00872$  bar

$$\omega_2 = 0.622 \frac{p_g}{p - p_g} = 0.0054$$

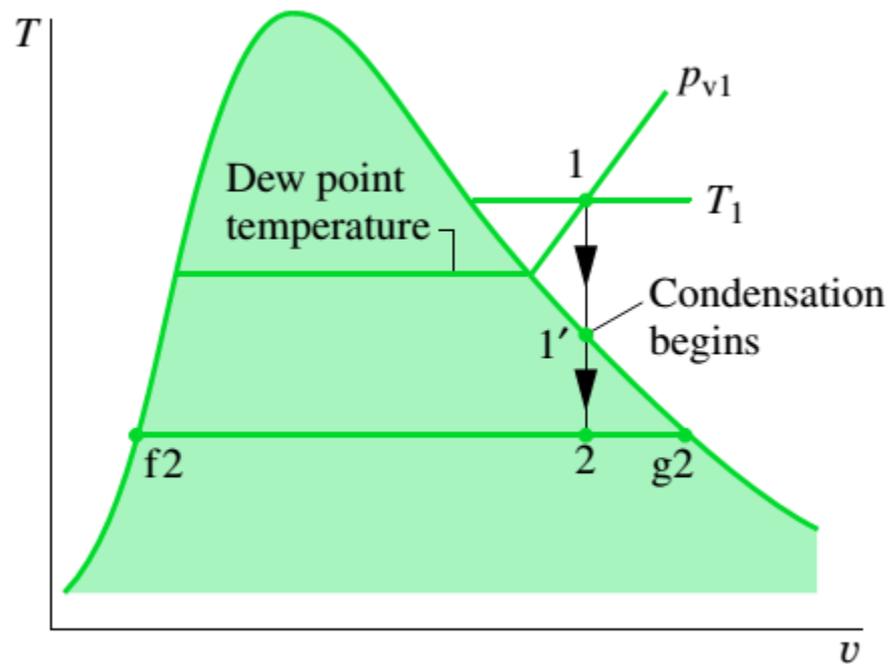
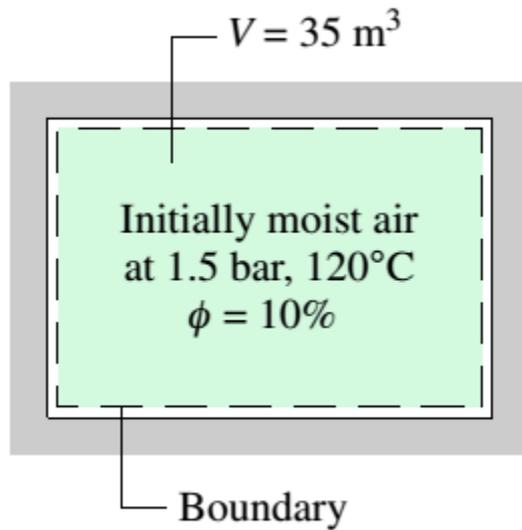
$$m_{v2} = \omega_2 m_a = (0.0054)(0.9891) = 0.0053 \text{ kg}$$

- The total condensate

$$m_w = m_{v1} - m_{v2} = 0.0109 - 0.0053 = 0.0056 \text{ kg}$$

# Example 2

- Cooling Moist Air at Constant Volume
- An air–water vapor mixture is contained in a rigid, closed vessel with a volume of  $35 \text{ m}^3$  at 1.5 bar,  $120^\circ\text{C}$ , and  $\phi = 10\%$ . The mixture is cooled at constant volume until its temperature is reduced to  $22^\circ\text{C}$ . Determine
- (a) the dew point temperature corresponding to the initial state, in  $^\circ\text{C}$ ,
- (b) the temperature at which condensation actually begins, in  $^\circ\text{C}$ , and
- (c) the amount of water condensed, in kg.



- The dew point temperature at the initial state is the saturation temperature corresponding to the partial pressure  $p_{v1}$ .  $p_{v1} = \phi_1 p_{g1} = (0.10)(1.985) = 0.1985 \text{ bar}$
- Interpolating, gives the dew point temperature as 60°C, which is the temperature condensation would begin if the moist air were cooled at constant pressure.

- (b) In the process from state 1 to state 1', the water exists as a vapor only. For the process from state 1' to state 2, the water exists as a two-phase liquid–vapor mixture. Note that pressure does not remain constant during the cooling process from state 1 to state 2.
- State 1' on the T–v diagram denotes the state where the water vapor first becomes saturated. The saturation temperature at this state is denoted as  $T'$ .
- Cooling to a temperature less than  $T'$  would result in condensation of some of the water vapor present. Since state 1' is a saturated vapor state, the temperature  $T'$  can be found by interpolating.

- The specific volume of the vapor at state 1 equals the specific volume of the vapor at state 1, which can be evaluated from the ideal gas equation

$$v_{v1} = \frac{(R / M_v) T_1}{p_{v1}} = \left( \frac{8314}{18} \right) \left( \frac{393K}{0.1985 \times 10^5} \right) = 9.145 \frac{m^3}{kg}$$

- Interpolation with  $v_{v1} = v_g$  gives  $T = 56^\circ\text{C}$ .
- (c) The amount of condensate equals the difference between the initial and final amounts of water vapor present. The mass of the water vapor present initially is

$$m_{v1} = \frac{V}{v_{v1}} = \frac{35}{9.145} = 3.827 \text{ kg}$$

- At the final state, the water forms a two-phase liquid–vapor mixture having a specific volume of  $9.145 \text{ m}^3/\text{kg}$ .
- the quality  $x_2$  of the liquid–vapor mixture can be

$$x_2 = \frac{v_{v2} - v_{f2}}{v_{g2} - v_{f2}} = \frac{9.145 - 1.0022 \times 10^{-3}}{51.447 - 1.0022 \times 10^{-3}} = 0.178$$

- where  $v_{f2}$  and  $v_{g2}$  are the saturated liquid and saturated vapor specific volumes at  $T_2=22^\circ\text{C}$ , respectively.
- the mass of the water vapor contained in the system at the final state is

$$m_{v2} = (0.178)(3.827) = 0.681 \text{ kg}$$

- The mass of the condensate,  $m_{w2}$ , is then

$$m_{w2} = m_{v1} - m_{v2} = 3.827 - 0.681 = 3.146 \text{ kg}$$

# Example 3

- Evaluating Heat Transfer for Moist Air Cooling at Constant Volume
- An air–water vapor mixture is contained in a rigid, closed vessel with a volume of  $35 \text{ m}^3$  at 1.5 bar,  $120^\circ\text{C}$ , and  $\phi = 10\%$ . The mixture is cooled until its temperature is reduced to  $22^\circ\text{C}$ . Determine the heat transfer during the process, in kJ.

- Solution
- closed system energy balance

$$\Delta U = Q - \cancel{W}$$

$$Q = U_2 - U_1$$

- where

$$U_1 = m_a u_{a1} + m_{v1} u_{v1} = m_a u_{a1} + m_{v1} u_{g1}$$

$$U_2 = m_a u_{a2} + m_{v2} u_{v2} + m_{w2} u_{w2} = m_a u_{a2} + m_{v2} u_{g2} + m_{w2} u_{f2}$$

- The specific internal energy of the water vapor at the initial state can be approximated as the saturated vapor value at  $T_1$ . At the final state, the water vapor is assumed to exist as a saturated vapor, so its specific internal energy is  $u_g$  at  $T_2$ . The liquid water at the final state is saturated, so its specific internal energy is  $u_f$  at  $T_2$ .

- Therefore,

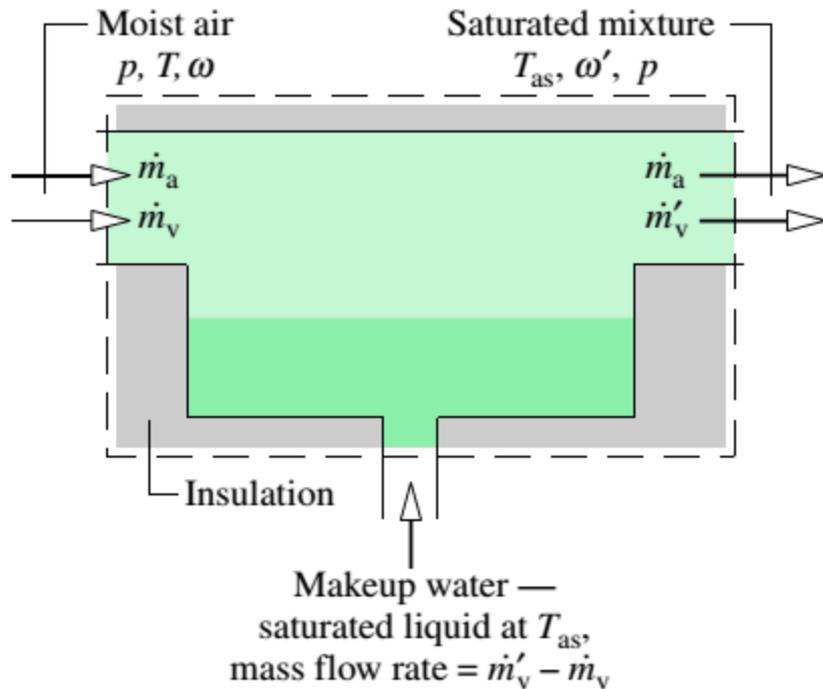
$$Q = m_a(u_{a2} - u_{a1}) + m_{v2}u_{g2} + m_{w2}u_{f2} - m_{v1}u_{g1}$$

- The mass of dry air,  $m_a$ , can be found using the ideal gas equation with the partial pressure of the dry air at the initial state obtained using  $p_{v1}=0.1985$  bar.

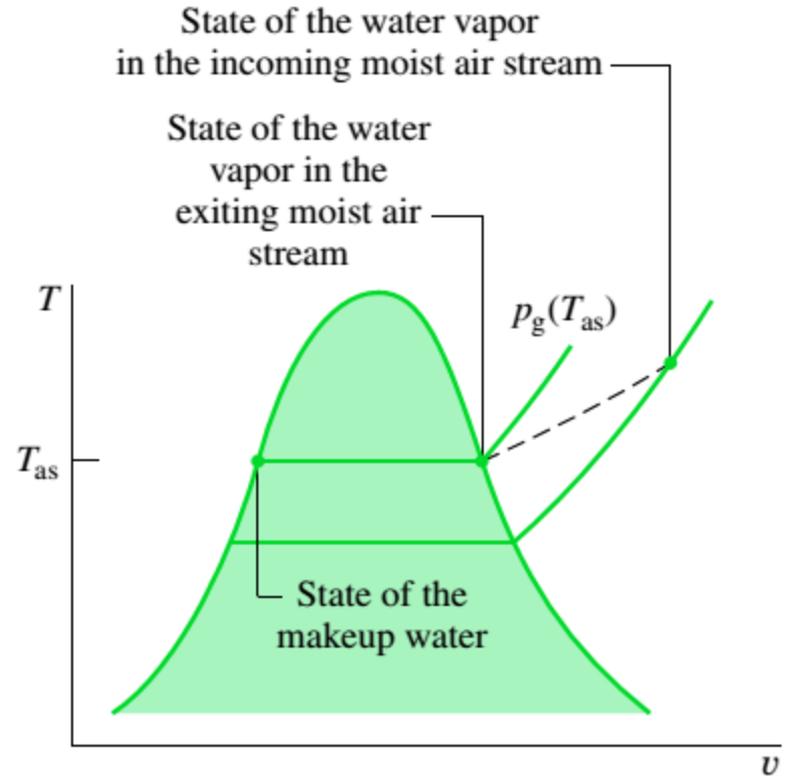
$$m_a = \frac{p_{a1}V}{(R/M_a)T_1} = \frac{(p - p_{v1})V}{(R/M_a)T_1} = \frac{(1.5 - 0.1985) \times 10^5 (35)}{(8314/28.97)(393)} = 40.398 \text{ kg}$$

- $Q = -10,603 \text{ kJ}$

# Modeling an Adiabatic Saturation Process



(a)



(b)

- The device is assumed to operate at steady state and adiabatic.
- An air–water vapor mixture of unknown humidity ratio enters the device at a known pressure  $p$  and temperature  $T$ .
- As the mixture passes through the device, it comes into contact with a pool of water. If the entering mixture is not saturated ( $\phi < 100\%$ ), some of the water would evaporate.
- The energy required to evaporate the water would come from the moist air, so the mixture temperature would decrease as the air passes through the duct.
- For a sufficiently long duct, the mixture would be saturated as it exits ( $\phi = 100\%$ ).
- The temperature of the exiting mixture is the **adiabatic-saturation temperature**.

- A steady flow of makeup water at temperature  $T_{as}$  is added at the same rate at which water is evaporated.

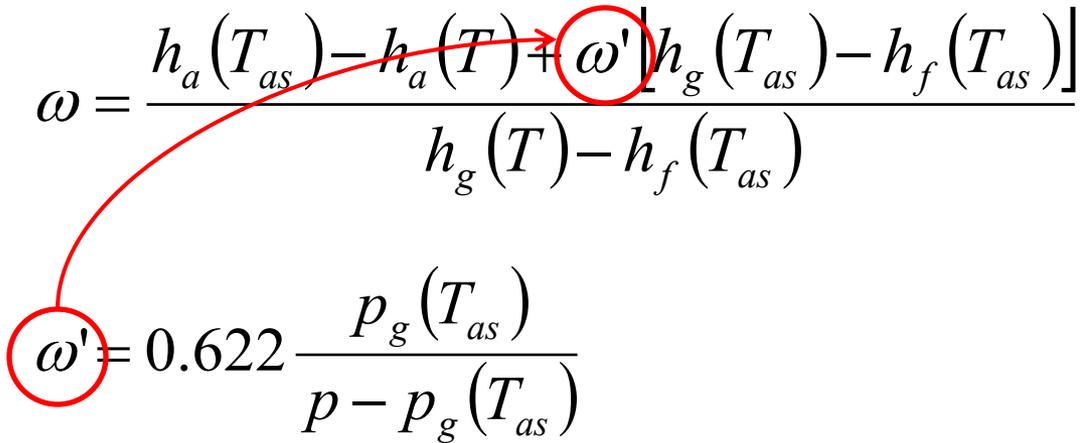
$$\begin{aligned} & \left( \dot{m}_a h_a(T) + \dot{m}_v h_v(T) \right)_{\text{moist air entering}} + \left( \left( \dot{m}'_v - \dot{m}_v \right) h_w(T_{as}) \right)_{\text{makeup water}} \\ & = \left( \dot{m}_a h_a(T_{as}) + \dot{m}'_v h_v(T_{as}) \right)_{\text{moist air exiting}} \end{aligned}$$

- Dividing by the mass flow rate of the dry air, the energy rate balance can be written on the basis of a unit mass of dry air passing through the device as,

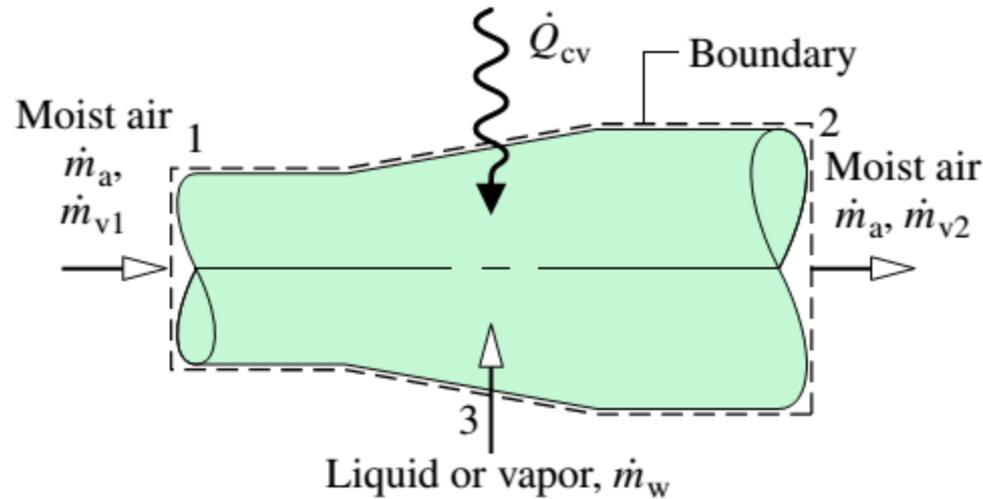
$$\begin{aligned} & \left( h_a(T) + \omega h_g(T) \right)_{\text{moist air entering}} + \left( (\omega' - \omega) h_f(T_{as}) \right)_{\text{makeup water}} \\ & = \left( h_a(T_{as}) + \omega' h_g(T_{as}) \right)_{\text{moist air exiting}} \end{aligned}$$

- where  $\omega = \dot{m}_v / \dot{m}_a$  and  $\omega' = \dot{m}'_v / \dot{m}_a$

- therefore

$$\omega = \frac{h_a(T_{as}) - h_a(T) + \omega' [h_g(T_{as}) - h_f(T_{as})]}{h_g(T) - h_f(T_{as})}$$
$$\omega' = 0.622 \frac{p_g(T_{as})}{p - p_g(T_{as})}$$


- Applying Mass and Energy Balances to Air-Conditioning Systems



- Mass balance  $\dot{m}_{a1} = \dot{m}_{a2}$  (*dry air*)  
 $\dot{m}_{v1} + \dot{m}_w = \dot{m}_{v2}$  (*water*)
- Or  $\dot{m}_w = \dot{m}_a(\omega_2 - \omega_1)$  (*water*)
- where  $\dot{m}_{v1} = \omega_1 \dot{m}_a$  and  $\dot{m}_{v2} = \omega_2 \dot{m}_a$

- energy balance,

$$0 = \dot{Q} + (\dot{m}_a h_{a1} + \dot{m}_{v1} h_{v1}) + \dot{m}_w h_w - (\dot{m}_a h_{a2} + \dot{m}_{v2} h_{v2})$$

- with  $\dot{m}_{v1} = \omega_1 \dot{m}_a$  and  $\dot{m}_{v2} = \omega_2 \dot{m}_a$

$$0 = \dot{Q} + \dot{m}_a (h_{a1} + \omega_1 h_{g1}) + \dot{m}_w h_w - \dot{m}_a (h_{a2} + \omega_2 h_{g2})$$

- Substitute mass balance

$$\dot{m}_w = \dot{m}_a (\omega_2 - \omega_1) \quad (\text{water})$$

- into, we have,

$$0 = \dot{Q} + \dot{m}_a (h_{a1} - h_{a2}) + \omega_1 h_{g1} + (\omega_2 - \omega_1) h_w - \omega_2 h_{g2}$$

Ideal gas mixture

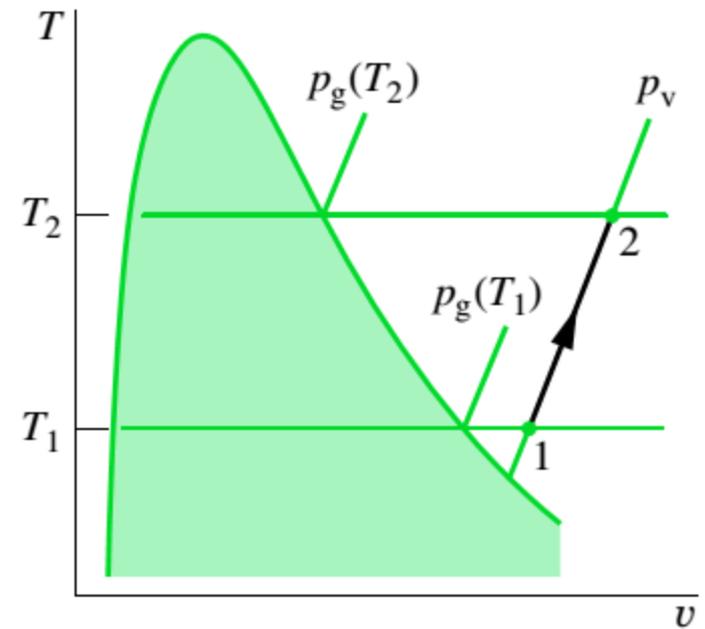
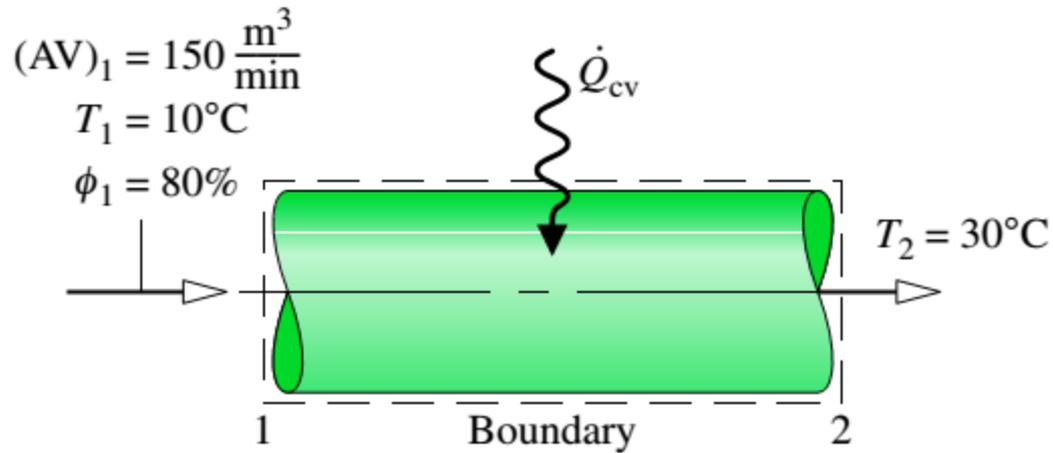
Using steam table

- where  $h_{a1} - h_{a2} = c_{pa} (T_1 - T_2)$

- Evaluating the enthalpies of the **water vapor** as the **saturated vapor** values at the respective temperatures and the enthalpy of each **liquid stream** as the **saturated liquid** enthalpy at the respective temperature.
- Because ...

# Example 1

- Heating Moist Air in a Duct
- Moist air enters a duct at  $10^{\circ}\text{C}$ , 80% relative humidity, and a volumetric flow rate of  $150\text{ m}^3/\text{min}$ . The mixture is heated as it flows through the duct and exits at  $30^{\circ}\text{C}$ . No moisture is added or removed, and the mixture pressure remains approximately constant at 1 bar. For **steady-state operation**, determine
- (a) the rate of heat transfer, in  $\text{kJ}/\text{min}$ , and
- (b) the relative humidity at the exit. Changes in kinetic and potential energy can be ignored.



- (a) Mass balance

$$\dot{m}_{a1} = \dot{m}_{a2} \quad (\text{dry air})$$

$$\dot{m}_{v1} = \dot{m}_{v2} \quad (\text{water})$$

- Energy balance

$$0 = \dot{Q} + (\dot{m}_a h_{a1} + \dot{m}_{v1} h_{v1}) - (\dot{m}_a h_{a2} + \dot{m}_{v2} h_{v2})$$

- Solving for Q

$$\begin{aligned}\dot{Q} &= \dot{m}_a(h_{a2} - h_{a1}) + \dot{m}_v(h_{v2} - h_{v1}) \\ &= \dot{m}_a[(h_{a2} - h_{a1}) + \omega(h_{v2} - h_{v1})]\end{aligned}$$

$$\dot{m}_a = \frac{(AV)_1}{v_{a1}}$$

$$v_{a1} = \frac{(R/M)T_1}{p_{a1}}$$

$$p_{v1} = \phi_1 p_{g1} = (0.8)(0.01228 \text{ bar}) = 0.0098 \text{ bar}$$

$$p_{a1} = 1 - p_{v1} = 0.9902 \text{ bar}$$

$$v_{a1} = \frac{\left(\frac{8314}{28.97}\right)(283)}{(0.9902 \times 10^5)} = 0.82 \text{ m}^3 / \text{kg}$$

$$\dot{m}_a = \frac{150}{0.82} = 182.9 \text{ kg / min}$$

$$\begin{aligned}\omega &= 0.622 \left( \frac{p_{v1}}{p - p_{v1}} \right) = 0.622 \left( \frac{0.0098}{1 - 0.0098} \right) \\ &= 0.00616 \frac{\text{kg(vapor)}}{\text{kg(dry air)}}\end{aligned}$$

- Finally

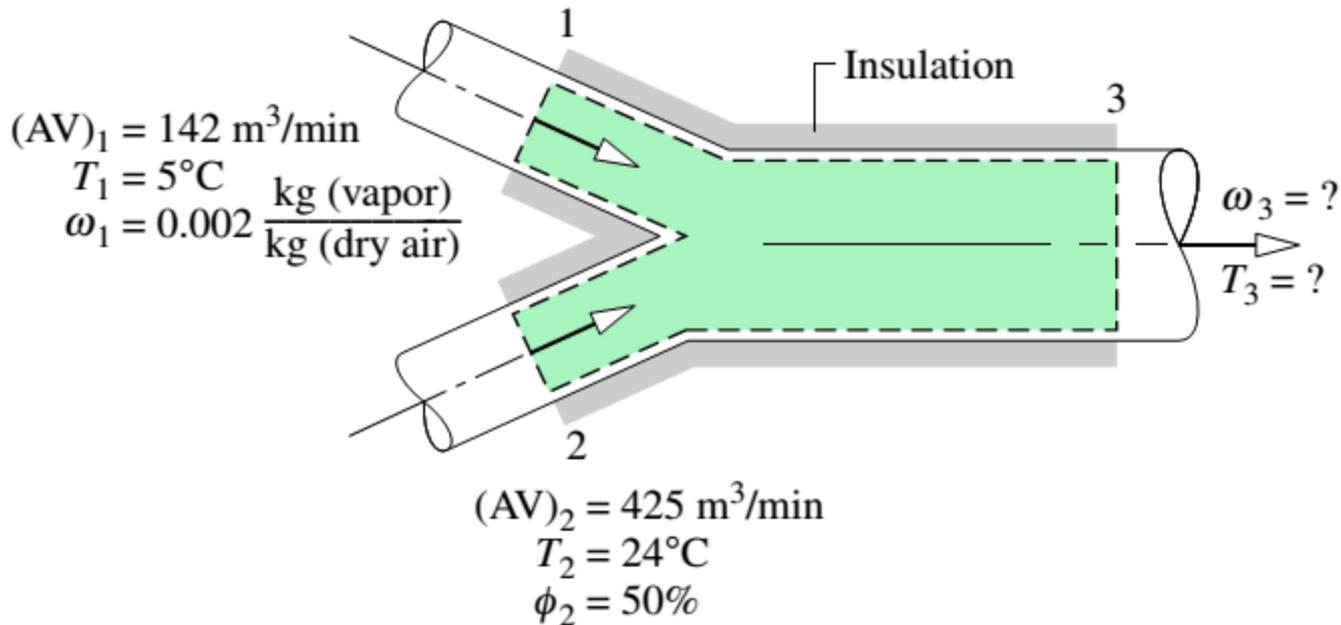
$$\begin{aligned}\dot{Q} &= \dot{m}_a [(h_{a2} - h_{a1}) + \omega(h_{v2} - h_{v1})] \\ &= (182.9)[(303.2 - 283.1) + 0.00616(2556.3 - 2519.8)] \\ &= 3717 \text{ kJ / min}\end{aligned}$$

- (b)

$$\phi_2 = \frac{p_{v2}}{p_{g2}} = \frac{0.0098}{0.04246} = 0.231$$

# Example 2

- Adiabatic Mixing of Moist Streams
- A stream consisting of  $142 \text{ m}^3/\text{min}$  of moist air at a temperature of  $5^\circ\text{C}$  and a humidity ratio of  $0.002 \text{ kg(vapor)/kg(dry air)}$  is mixed adiabatically with a second stream consisting of  $425 \text{ m}^3/\text{min}$  of moist air at  $24^\circ\text{C}$  and 50% relative humidity. The pressure is constant throughout at 1 bar. Using the psychrometric chart, determine
- (a) the humidity ratio and
- (b) the temperature of the exiting mixed stream, in  $^\circ\text{C}$ .



- Solution
- (a) The humidity ratio  $\omega_3$  can be found by means of mass rate balances for the dry air and water vapor, respectively

$$\dot{m}_{a1} + \dot{m}_{a2} = \dot{m}_{a3} \quad (\text{dry air})$$

$$\dot{m}_{v1} + \dot{m}_{v2} = \dot{m}_{v3} \quad (\text{water})$$

• Let  $\dot{m}_{v1} = \omega_1 \dot{m}_{a1}$ ,  $\dot{m}_{v2} = \omega_2 \dot{m}_{a2}$  and  $\dot{m}_{v3} = \omega_3 \dot{m}_{a3}$

• Then  $\omega_1 \dot{m}_{a1} + \omega_2 \dot{m}_{a2} = \omega_3 \dot{m}_{a3}$  (water)

$$\omega_3 = \frac{\omega_1 \dot{m}_{a1} + \omega_2 \dot{m}_{a2}}{\dot{m}_{a3}}$$

• Since  $\dot{m}_{a3} = \dot{m}_{a1} + \dot{m}_{a2}$

$$\omega_3 = \frac{\omega_1 \dot{m}_{a1} + \omega_2 \dot{m}_{a2}}{\dot{m}_{a1} + \dot{m}_{a2}}$$

• Then  $\dot{m}_{a1} = \frac{(AV)_1}{v_{a1}}$ ,  $\dot{m}_{a2} = \frac{(AV)_2}{v_{a2}}$

• The values of  $v_{a1}$ , and  $v_{a2}$ , and  $\omega_2$  are readily found from the psychrometric chart.

- Thus, at  $\omega_1 = 0.002$  and  $T_1 = 5^\circ\text{C}$ ,  $v_{a1} = 0.79 \text{ m}^3/\text{kg}(\text{dry air})$ . At  $\phi_2 = 50\%$  and  $T_2 = 24^\circ\text{C}$ ,  $v_{a2} = 0.855 \text{ m}^3/\text{kg}(\text{dry air})$  and  $\omega_2 = 0.0094$ .
- The mass flow rates of the dry air are then  $\dot{m}_{a1} = 180 \text{ kg}(\text{dry air})/\text{min}$  and  $\dot{m}_{a2} = 497 \text{ kg}(\text{dry air})/\text{min}$ ,

$$\omega_3 = \frac{(0.002)(180) + (0.0094)(497)}{180 + 497} = 0.0074 \frac{\text{kg}(\text{vapor})}{\text{kg}(\text{dry air})}$$

- (b) The temperature  $T_3$  of the exiting mixed stream can be found from an energy rate balance.

$$\dot{m}_{a1}(h_a + \omega h_v)_1 + \dot{m}_{a2}(h_a + \omega h_v)_2 = \dot{m}_{a3}(h_a + \omega h_v)_3$$

- From the table,

$$(h_a + \omega h_v)_1 = 10 \text{ kJ} / \text{kg}(\text{dry air})$$

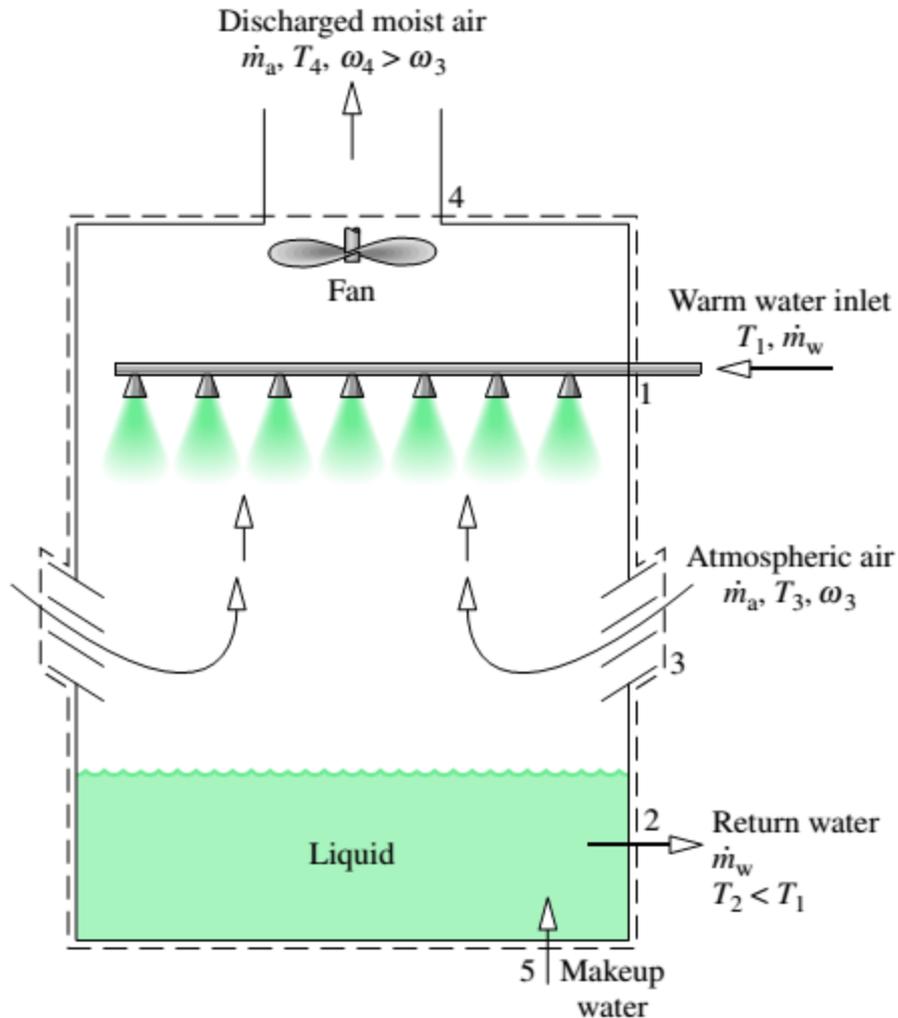
$$(h_a + \omega h_v)_2 = 47.8 \text{ kJ} / \text{kg}(\text{dry air})$$

- Therefore

$$\begin{aligned}(h_a + \omega h_v)_3 &= \frac{\dot{m}_{a1}(h_a + \omega h_v)_1 + \dot{m}_{a2}(h_a + \omega h_v)_2}{\dot{m}_{a1} + \dot{m}_{a2}} \\ &= \frac{180(10) + 497(47.8)}{180 + 497} = 37.7 \frac{\text{kJ}}{\text{kg}(\text{dry air})}\end{aligned}$$

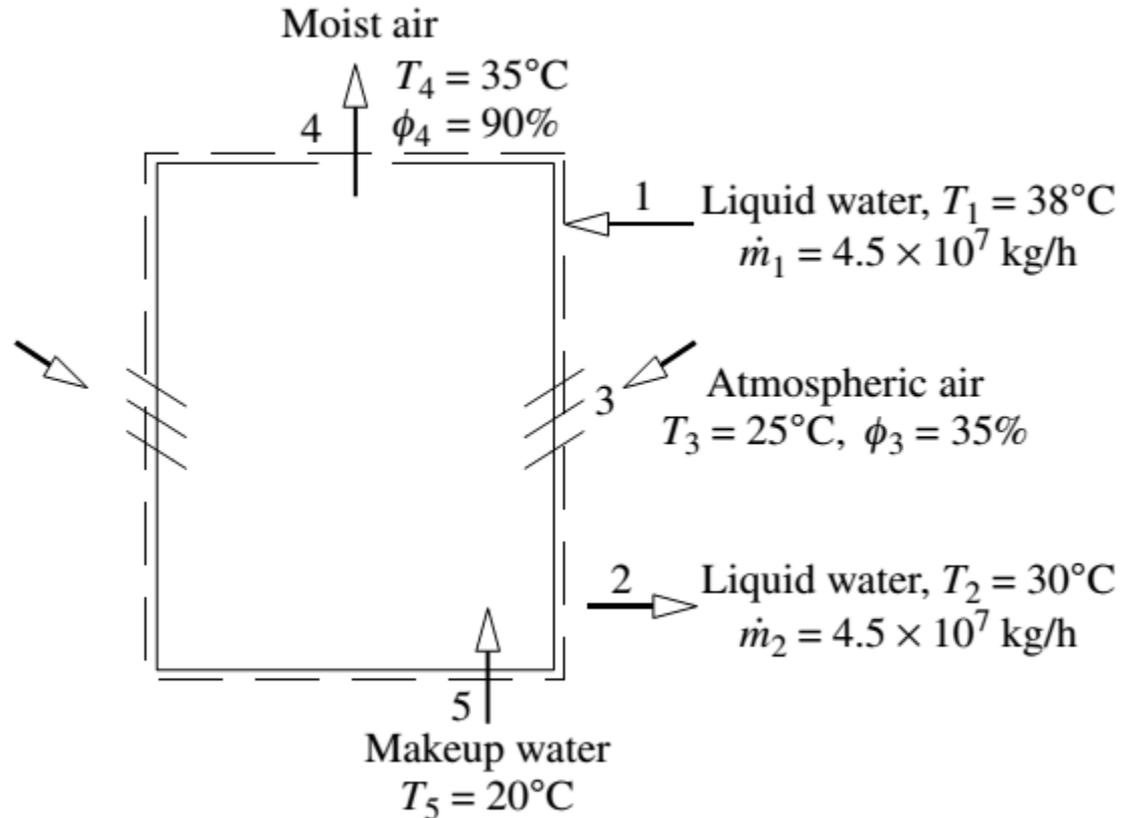
- Using this value for the enthalpy of the moist air at the exit, together with the previously determined value for  $\omega_3$ , fixes the state of the exiting moist air. From inspection of table,  $T_3 = 19^\circ\text{C}$ .

# Cooling Towers



- Forced-convection, counterflow cooling tower

# Example



- Determine the mass flow rates of the dry air and the makeup water, in kg/h.

- Mass balance

$$\dot{m}_{a3} = \dot{m}_{a4} \quad (\text{dry air})$$

$$\cancel{\dot{m}_1} + \dot{m}_5 + \dot{m}_{v3} = \cancel{\dot{m}_2} + \dot{m}_{v4} \quad (\text{water})$$

- Let  $\dot{m}_{v3} = \omega_3 \dot{m}_a$  and  $\dot{m}_{v4} = \omega_4 \dot{m}_a$

$$\dot{m}_5 = \dot{m}_a (\omega_4 - \omega_3)$$

- Energy balance

$$0 = \dot{m}_1 h_{w1} + (\dot{m}_a h_{a3} + \dot{m}_{v3} h_{v3}) + \dot{m}_5 h_{w5} - \dot{m}_2 h_{w2} - (\dot{m}_a h_{a4} + \dot{m}_{v4} h_{v4})$$

- Evaluating the enthalpies of the water vapor as the saturated vapor values at the respective temperatures and the enthalpy of each liquid stream as the saturated liquid enthalpy at the respective temperature,

- the energy rate equation becomes,

$$0 = \dot{m}_1 h_{f1} + (\dot{m}_a h_{a3} + \dot{m}_{v3} h_{g3}) + \dot{m}_5 h_{f5} - \dot{m}_2 h_{f2} - (\dot{m}_a h_{a4} + \dot{m}_{v4} h_{g4})$$

- Let  $\dot{m}_1 = \dot{m}_2$ ,  $\dot{m}_{v3} = \omega_3 \dot{m}_a$  and  $\dot{m}_{v4} = \omega_4 \dot{m}_a$ ,  $\dot{m}_5 = \dot{m}_a (\omega_4 - \omega_3)$

- Then 
$$\dot{m}_a = \frac{\dot{m}_1 (h_{f1} - h_{f2})}{h_{a4} - h_{a3} + \omega_4 h_{g4} - \omega_3 h_{g3} - (\omega_4 - \omega_3) h_{f5}}$$

- The humidity ratios  $\omega_3$  and  $\omega_4$  can be determined using the partial pressure of the water vapor obtained with the respective relative humidity

$$\omega_3 = 0.00688 \text{ and } \omega_4 = 0.0327,$$

- Therefore,  $\dot{m}_a = 2.03 \times 10^7 \text{ kg/h}$

$$\dot{m}_5 = (2.03 \times 10^7)(0.0327 - 0.00688) = 5.24 \times 10^5 \text{ kg/h}$$