Fluid Systems

(Understanding Engineering Thermo—Octave Levenspiel)

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Two points I want to emphasis:

1. ΔU and ΔH
2. $W = W_{pv} + W_{sh}$

BATCH OF IDEAL GAS

Batch of ideal gas

- Constant volume

$$v_1 = v_2 \quad and \quad \frac{p_1}{T_1} = \frac{p_2}{T_2}$$

$$w_{rev} = \int p dv = 0$$

$$q_{rev} = \Delta u + w_{rev} = c_v \Delta T + 0 = c_v \Delta T \ \text{ J/mol}$$

- Constant pressure

$$p_1 = p_2 \quad and \quad \frac{v_1}{T_1} = \frac{v_2}{T_2}$$

$$w_{rev} = \int p dv = p(v_2 - v_1) = p_1 v_1 \left(\frac{T_2}{T_1} - 1 \right) = \frac{p_1 v_1}{T_1} (T_2 - T_1) = R \Delta T$$

$$q_{rev} = \Delta u + w_{rev} = c_v \Delta T + R \Delta T = c_p \Delta T \ \text{ J/mol}$$

Batch of ideal gas

- Constant temperature

$$T_1 = T_2 \quad and \quad p_1 v_1 = p_2 v_2$$

$$\Delta h = \underbrace{\Delta u}_{0} + \underbrace{\Delta(pv)}_{0} = 0$$

$$q_{rev} = w_{rev} = \int p dv = \int \frac{RT}{v} dv$$

$$= RT \ln \frac{v_2}{v_1} = RT \ln \frac{p_1}{p_2} \quad \text{J/mol}$$

Batch of ideal gas

- Adiabatic (q=0) reversible process with constant c_v

$$\underset{c_v dT}{du} = \overset{0}{dq_{rev}} - dw_{rev} = \underset{\frac{RT}{v}\,dv}{-pdv}$$

- Integrate

$$\int_{T_1}^{T_2} \frac{dT}{T} = -\frac{R}{c_v}\int_{v_1}^{v_2}\frac{dv}{v}$$

- Assume constant c_v, hence constant c_p,
- Introduce symbol k,

$$k = \frac{c_p}{c_v} = 1 + \frac{R}{c_v}$$

- On integration,

$$\ln \frac{T_2}{T_1} = -(k-1)\ln \frac{v_2}{v_1}$$

- therefore

$$\boxed{\begin{aligned} \frac{T_2}{T_1} &= \left(\frac{v_1}{v_2}\right)^{k-1} \\ \frac{T_2}{T_1} &= \left(\frac{p_2}{p_1}\right)^{(k-1)/k} \\ \frac{p_2}{p_1} &= \left(\frac{v_1}{v_2}\right)^{k}, \quad or \;\; pv^{k} = \text{const.} \end{aligned}}$$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(k-1)/k}$$

- For ideal adiabatic reversible batch process, $\Delta e_p = \Delta e_k = 0$, $q_{rev} = 0$,

$$pv^k = \text{const.}$$

$$w_{rev} = \int p dv = \int_{v_1}^{v_2} \frac{const.}{v^k} dv$$

$$= \frac{p_1 v_1}{k-1}\left[1 - \left(\frac{p_2}{p_1}\right)^{(k-1)/k}\right] = \frac{RT_1}{k-1}\left[1 - \left(\frac{p_2}{p_1}\right)^{(k-1)/k}\right]$$

$$w_{rev} = -\Delta u = c_v (T_2 - T_1)$$

$$= -\frac{R}{k-1}(T_2 - T_1)$$

$$k = \frac{c_p}{c_v} = 1 + \frac{R}{c_v}$$

$$= -\frac{p_2 v_2 - p_1 v_1}{k-1}$$

$$pv = RT$$

$$= \frac{p_1 v_1}{k-1}\left(1 - \frac{p_2 v_2}{p_1 v_1}\right), \qquad \frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^k$$

$$dV^{q} = qV^{q-1}dV$$

$$-k = q - 1$$

$$dV^{1-k} = (1-k)V^{-k}dV$$

$$\frac{dV^{1-k}}{1-k} = V^{-k}dV$$

$$\frac{p_1 V_1^{K}}{1-k} dV^{1-k} = const \cdot V^{-k}dV$$

$$\boxed{pV^{k} = \text{const.}}$$

$$\int_{V_1}^{V_2} \frac{const}{V^k} dV = \frac{p_1 V_1^K}{1-k} \int_{V_1}^{V_2} dV^{1-k}$$

$$= \frac{p_1 V_1^K}{1-k} \left(V_2^{1-k} - V_1^{1-k} \right)$$

$$= \frac{p_1 V_1}{1-k} V_1^{k-1} \left(V_2^{1-k} - V_1^{1-k} \right)$$

$$= \frac{p_1 V_1}{k-1} \left(1 - \frac{V_2^{1-k}}{V_1^{1-k}} \right)$$

Recall

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{k-1}$$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(k-1)/k}$$

$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{k}, \quad or \quad pv^k = \text{const.}$$

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{k-1} = \left(\frac{p_2}{p_1}\right)^{(k-1)/k}$$

Example I

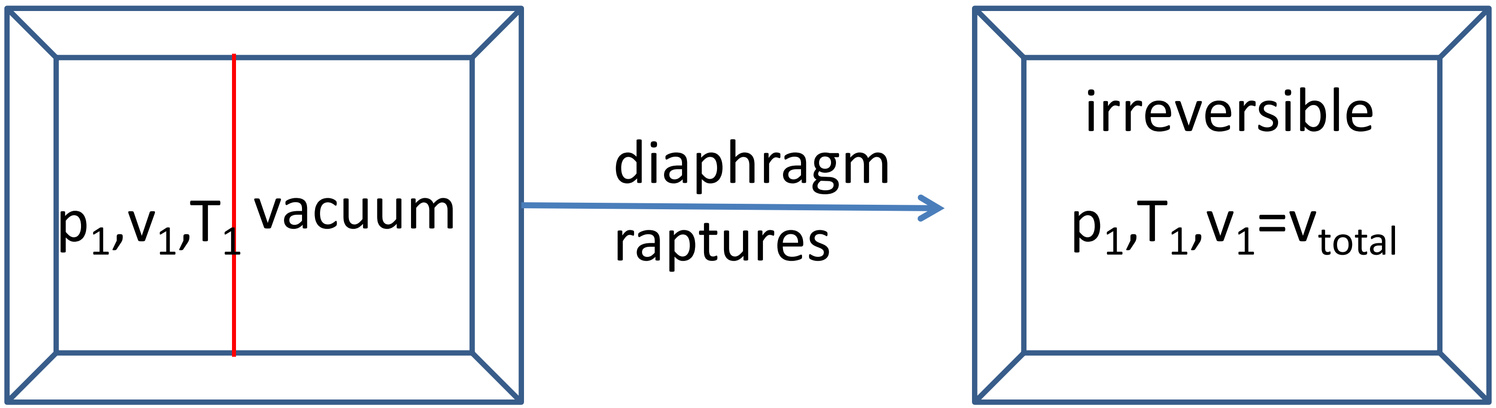
- Slow leak from an insulated tank

The gas remaining in the tank experiences and adiabatic reversible expansion, therefore

$$\boxed{\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(k-1)/k}}$$

Example II

- Rupture of a diaphragm in an insulated tank



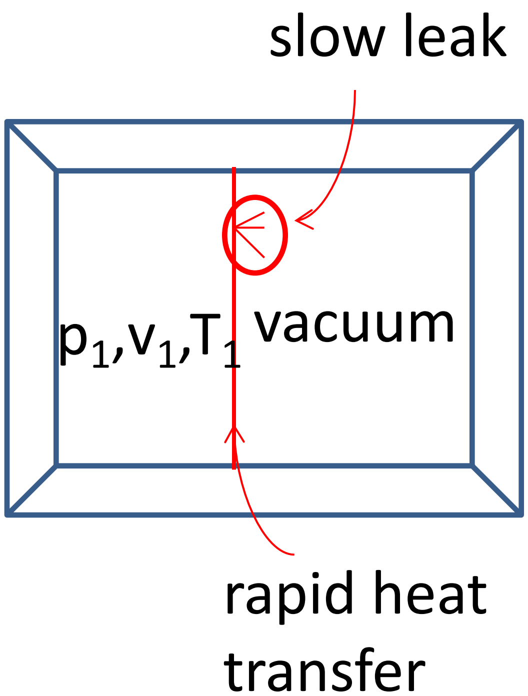
- Insulated and const volume

$$\Delta U = \cancelto{0}{Q} - \cancelto{0}{W}$$

$$T_2 = T_1 \quad and \quad \frac{p_2}{p_1} = \frac{V_2}{V_1}$$

Example III

- Slow leak between sections of an insulated tank



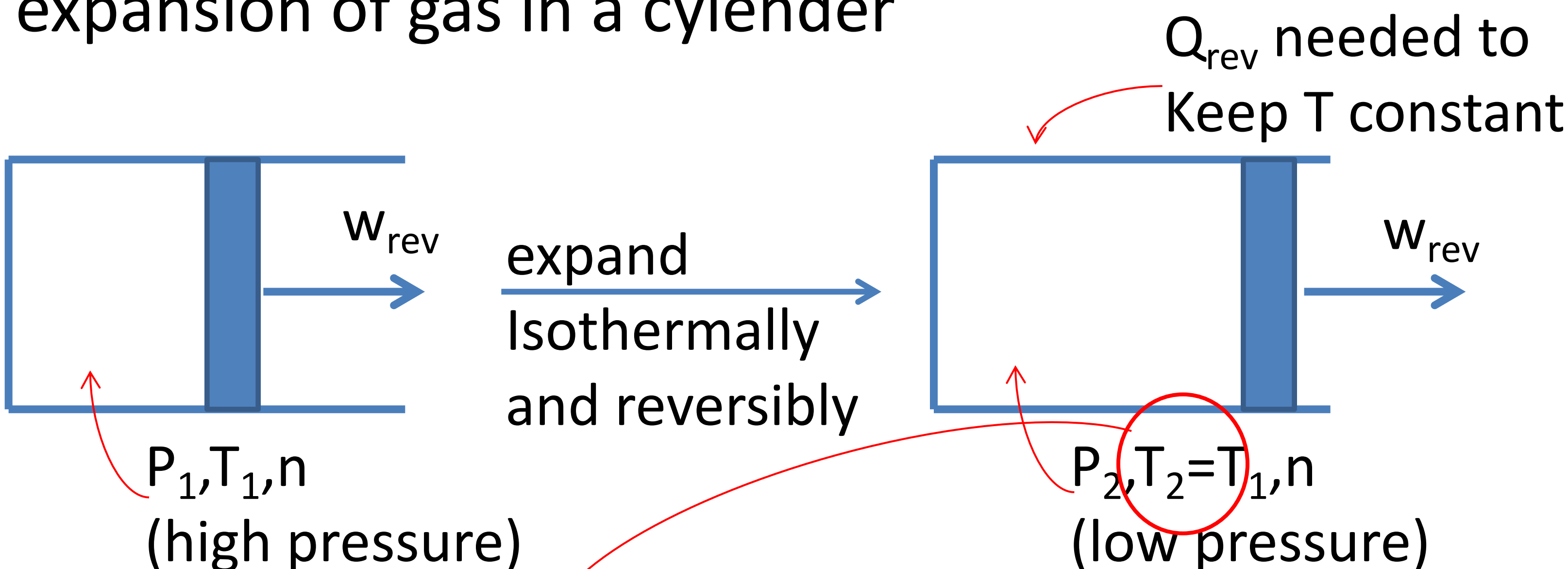
Insulated and const volume

$$\Delta U = \underbrace{\cancel{Q}}_{0} - \underbrace{\cancel{W}}_{0}$$

$$T_2 = T_1 \quad and \quad \frac{p_2}{p_1} = \frac{V_2}{V_1}$$

Example IV

- Heat involved in the slow isothermal reversible expansion of gas in a cylender

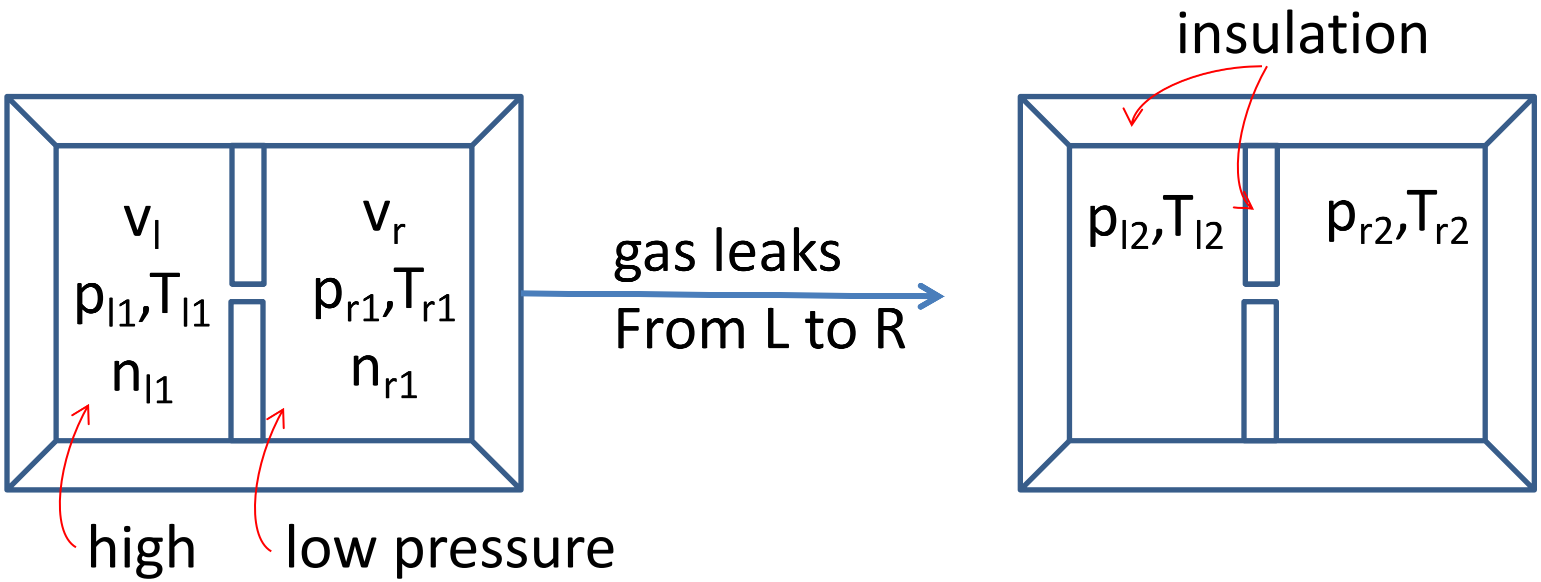


$$\boxed{\begin{aligned} 0 &= \Delta U = Q_{rev} - W_{rev} \\ Q_{rev} &= W_{rev} = nRT \ln \frac{p_1}{p_2} \end{aligned}}$$

constant temperature process

Example V

- Leak between two interconnected insulated tanks



- The gas remaining on the left-hand side expands adiabatically and reversibly.
- Find how T_l, T_r, p_l and p_r change

$$n_{l1} = \left(\frac{pv}{RT}\right)_{l1}$$

$$n_{r1} = \left(\frac{pv}{RT}\right)_{r1}$$

$n_{l1}+n_{r1} = n_{total}$

$$n_{l2} = \left(\frac{pv}{RT}\right)_{l2}$$

$$n_{r2} = \left(\frac{pv}{RT}\right)_{r2}$$

$n_{l2}+n_{r2} = n_{total}$

adiabatically and reversibly expansion,

$$\boxed{\frac{T_{l2}}{T_{l1}} = \left(\frac{p_{l2}}{p_{l1}}\right)^{(k-1)/k}}$$

$$\sum E_2 = \sum E_1 \ : (n_l u_l)_2 + (n_r u_r)_2 = (n_l u_l)_1 + (n_r u_r)_1$$

$$\boxed{(n_l T_l)_2 + (n_r T_r)_2 = (n_l T_l)_1 + (n_r T_r)_1}$$

$$\boxed{\begin{aligned} u &= c_v T \\ h &= c_p T \end{aligned}}$$

Example VI

- Total work done by an expanding gas

A 2 liter plastic pop bottle contains air at 300K and 1.5 bar gauge pressure. How much work could be done by this gas if you could expand it down to 1 bar

  - Isothermally and reversibly?
  - Adiabatically and reversibly?

$$n = \frac{pv}{RT} = \frac{(12.5 \times 15^{5})(0.002)}{(8.314)(300)} = 1.00 \text{ mol}$$

- Isothermal expansion

$$\boxed{W_{rev} = nRT \ln \frac{p_1}{p_2}}$$

$$W_{rev} = (1)(8.314)(300) \ln \frac{12.5}{1} = 6300 \text{ J}$$

- Adiabatic expansion

$$\boxed{W_{rev} = \frac{nRT_1}{k-1}\left[1 - \left(\frac{p_2}{p_1}\right)^{(k-1)/k}\right]}$$

$$W_{rev} = \frac{(1)(8.314)(300)}{1.4-1}\left[1 - \left(\frac{1}{12.5}\right)^{0.4/1.4}\right] = 3205 \text{ J}$$

Example VII

- Net work done by an expanding gas

The previous example calculated the work done by an expanding gas. However, in doing so the gas had to push back the 1 bar atmosphere. Let us now account for this work, subtract it from the work done, and thereby evaluate the useful work (shaft work) that could be extracted by this

  - Isothermal expansion
  - Adiabatic expansion

- Isothermal expansion
  - The work needed to push back the atmosphere is

$$W_{pv} = p_0(v_2 - v_1)$$
$$= (1 \times 10^5)(12.5 \times 0.002 - 0.002) = 2300 \text{ J}$$

  - The reversible shaft work that can be extracted is

$$\boxed{W_{sh} = 6300 - 2300 = 4000 \text{ J}}$$

- Adiabatic expansion
  - Final temperature of the expanded air is not 300K, but

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{(k-1)/k} = 300\left(\frac{1}{12.5}\right)^{0.4/1.4} = 146 \text{ K}$$

$$W_{pv} = p_0(v_2 - v_1)$$

$$= (1 \times 10^5)(12.5 \frac{146}{300} 0.002 - 0.002) = 1015 \text{ J}$$

  - The reversible shaft work that can be extracted is

$$\boxed{W_{sh} = 3205 - 1015 = 2190 \text{ J}}$$

Example VIII

- Explosion: The popping pop bottle
  - reversible or irreversible?
  - Isothermal or adiabatic?
- From the first law, and adiabatic

$$\Delta U = \overset{0}{Q} - W_1$$

- Highly irreversible, all go to push back

$$W_2 = \int p dv = p_{surr} \Delta v$$

- Therefore,

$$W_1 = \Delta U = nc_v(T_{initial} - T_{final})$$
$$W_2 = p_{surr}(v_{finall} - v_{initial})$$

- Then

$$W_1 = (1)(29.099 - 8.314)(300 - T_{final})$$
$$W_2 = 1 \times 10^5 \left[12.5 \times 0.002 \left(\frac{T_{final}}{300} \right) - 0.002 \right]$$

- Solving T_{final} by equating $W_1 = W_2$
- Finally

$$\boxed{\begin{aligned} T_{finall} &= 221 \text{ K} \\ \mathrm{W} &= 1642 \text{ J} \end{aligned}}$$

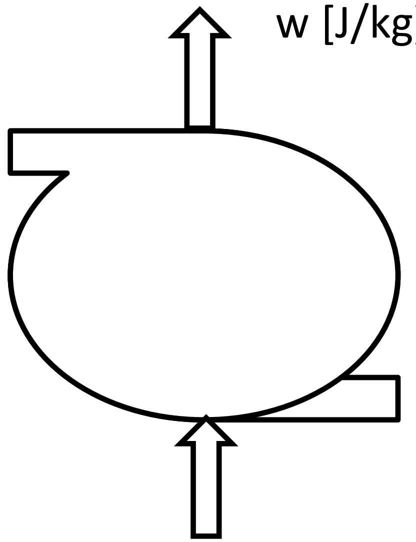
STEADY STATE FLOW SYSTEMS

Steady State Flow System

w [J/kg]

$e_1=u_1+e_{p1}+e_{k1}$ [J/kg]

$$\left(w_{pv,in} = \int_0^{v_1} p\,dv = p_1 v_1 \right)$$



$e_2=u_2+e_{p2}+e_{k2}$ [J/kg]

$$\left(w_{pv,out} = \int_0^{v_2} p\,dv = p_2 v_2 \right)$$

q [J/kg]

$$\underbrace{(\Delta U+\Delta E_p+\Delta E_k)}_{\Delta E} = Q - W$$

work done by system

$$w_{total} = w_{sh} + \overbrace{\int_0^{V_2} p\,dv}^{= p_2V_2} - \overbrace{\int_0^{V_1} p\,dv}^{= p_1V_1}$$

the rest work done by the system—shaft work

Steady State Flow System

- **Rearrange**

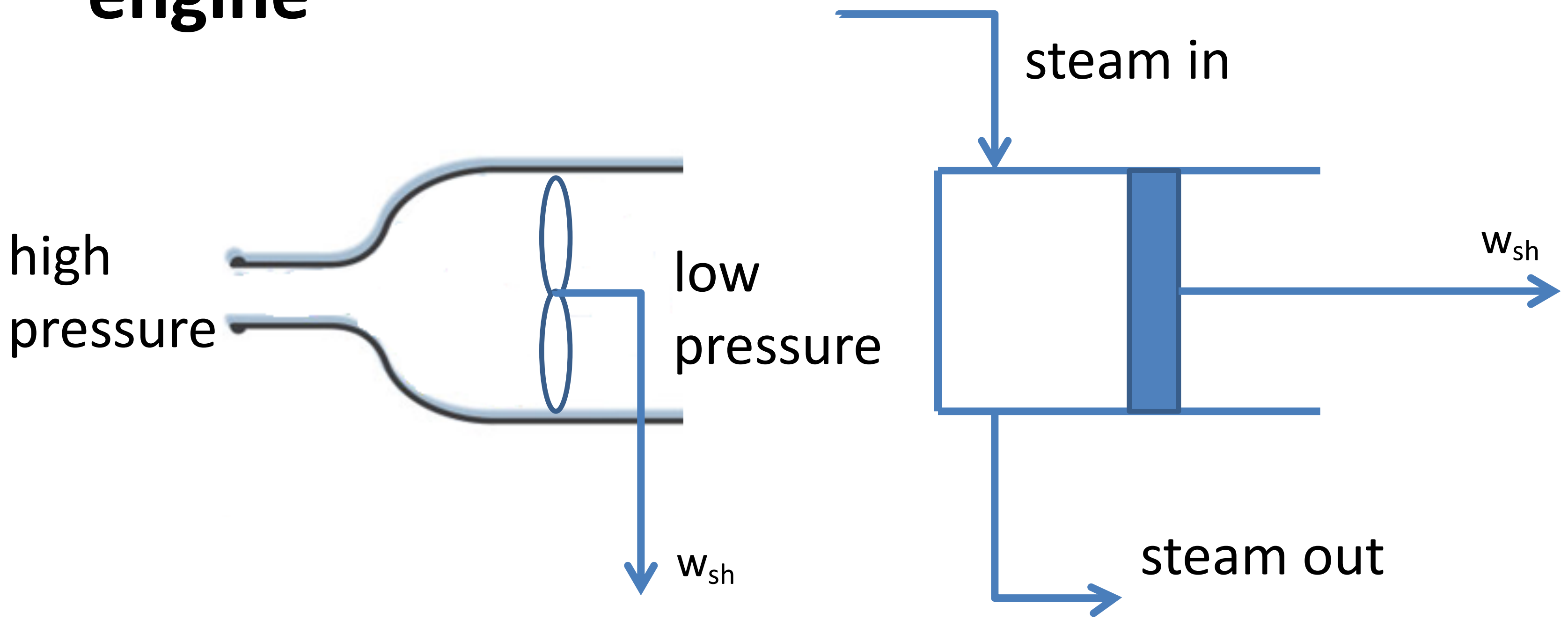
$$\underbrace{\underbrace{\left(u_2 + p_2 v_2\right)}_{h_2} - \underbrace{\left(u_1 + p_1 v_1\right)}_{h_1}}_{\Delta h} + g\Delta z + \overbrace{\frac{1}{2}\Delta v^2}^{0} = q - w_{sh}$$

for the flow streams,
not the system

$$\boxed{\begin{aligned} &RT_2 - RT_1 = R\Delta T \\ &CvT_2 - CvT_1 + RT_2 - RT_1 \\ &= (Cv + R)\Delta T = Cp\Delta T \end{aligned}}$$

Example I

- **The steam or water turbine and the steam engine**

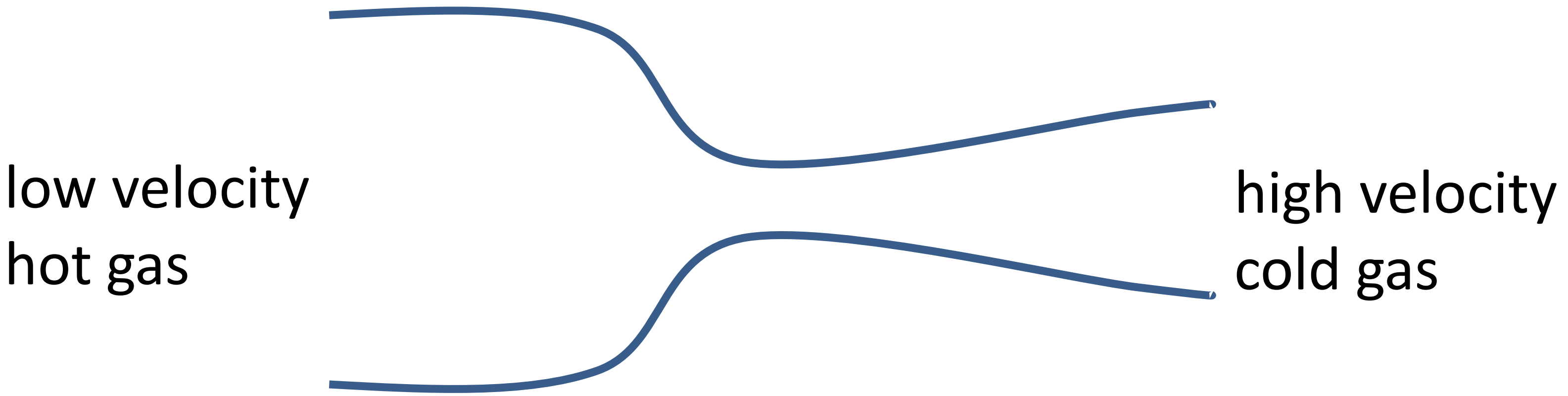


$$\dot{m}\Delta h = \dot{m}\left(h_2 - h_1\right) = -\dot{w}_{sh} \quad [W]$$

$$\Delta h = -w_{sh} \quad [J / kg]$$

Example II

- **The adiabatic flow nozzle**

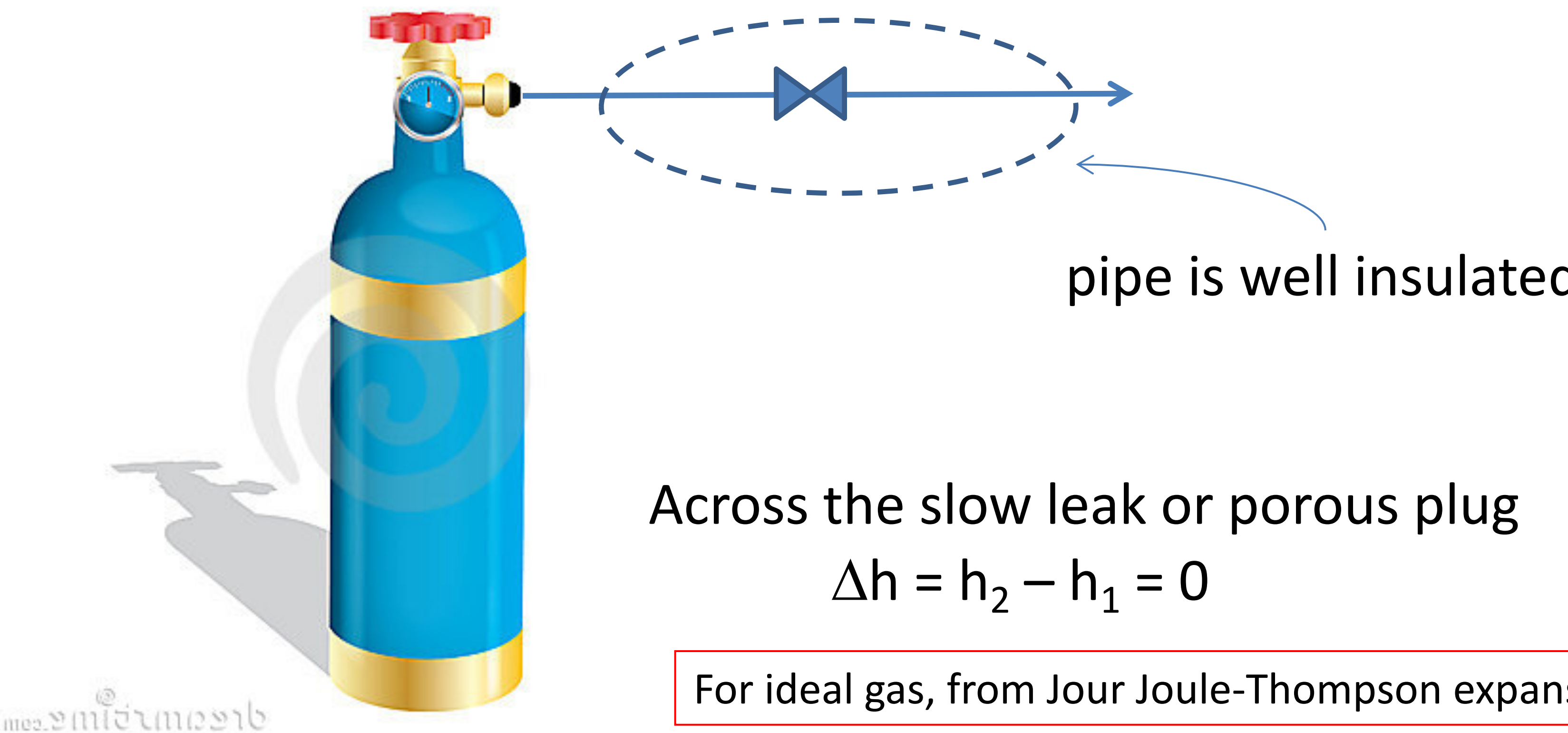


$$\dot{m}\Delta h = -\frac{\dot{m}}{2} \underline{V}_2^2 \qquad [W]$$

$$\Delta h = -\frac{1}{2} \underline{V}_2^2 \qquad [\frac{J}{kg}]$$

Example III

- **The Joule-Thomson expansion**



Across the slow leak or porous plug

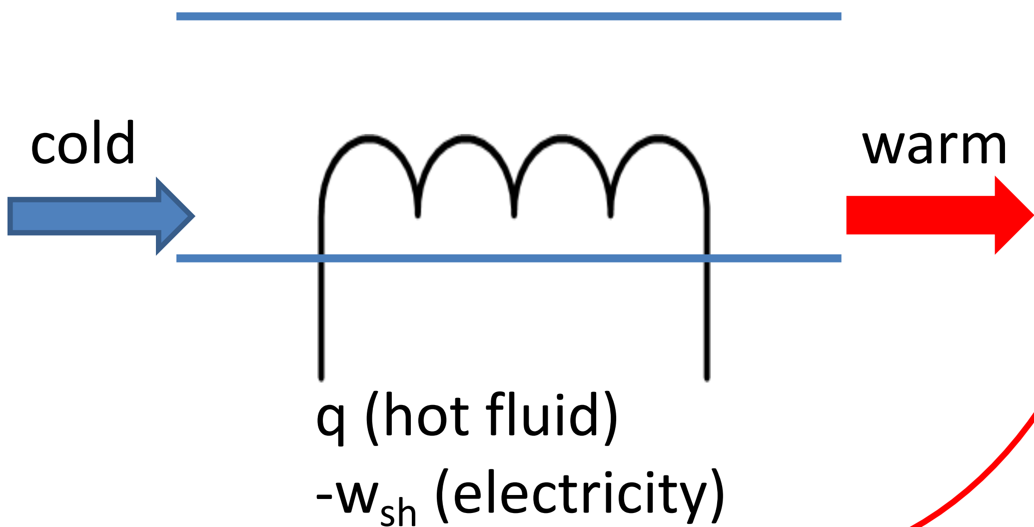
$$\Delta h = h_2 - h_1 = 0$$

For ideal gas, from Jour Joule-Thompson expansion

Example IV

- **The flow heater**

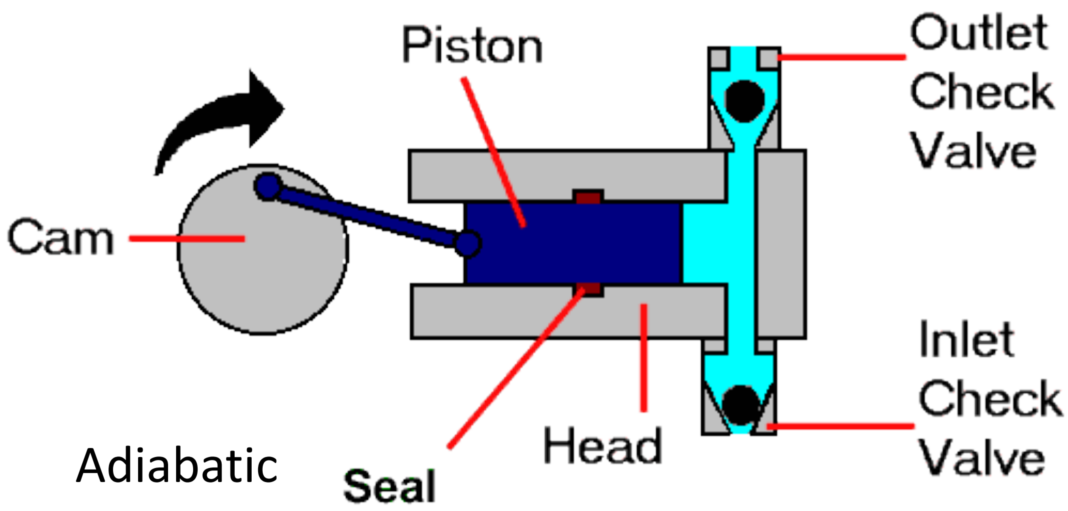
$$\boxed{\begin{aligned} &CvT_2 - CvT_1 + RT_2 - RT_1 \\ &= (Cv + R)\Delta T = Cp\Delta T \end{aligned}}$$



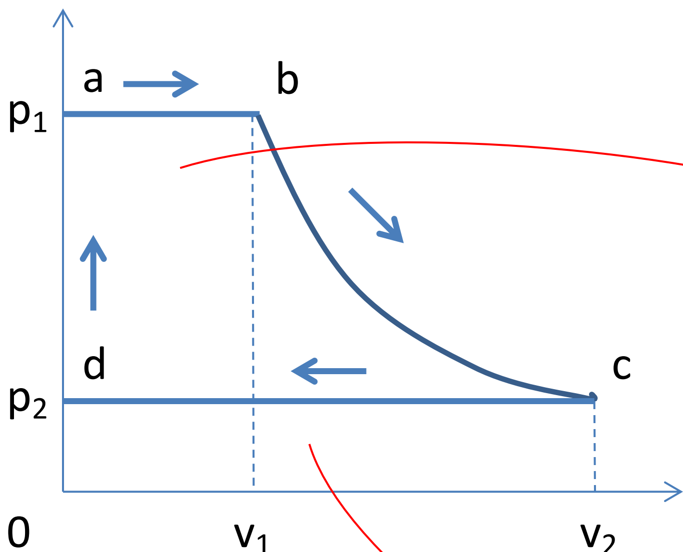
$$\left.\begin{aligned} \dot{m}\Delta h &= \dot{q} \\ \dot{m}\Delta h &= -\dot{W}_{sh} \end{aligned}\right\} \text{heat or work done on fluid in the heater}$$

Example V

- **Ideal piston-cylinder engine or ideal piston-cylinder pump**



Example V



- a-b Introduce 1 kg of high pressure gas at p_1 and of volume v_1.

$$w_1 = \int_0^{v_1} p_1 dv = p_1 v_1$$

- b-c Expand the gas to the outlet pressure p_2. (both valves are closed)

$$w_2 = \int_{v_1}^{v_2} p dv$$

- c-d Push out all the gas in the cylinder.

$$w_3 = \int_{v_2}^{0} p_2 dv = p_2 v_2$$

- Net shaft work done by the fluid

$$w_{sh} = w_1 + w_2 + w_3$$

$$= p_1 v_1 + \int_{v_1}^{v_2} p dv - p_2 v_2$$

$$\boxed{\begin{aligned} &CvT_2 - CvT_1 + RT_2 - RT_1 \\ &= (Cv + R)\Delta T = Cp\Delta T \end{aligned}}$$

- From the pv diagram

$$= -\int_{p_1}^{p_2} v dp$$

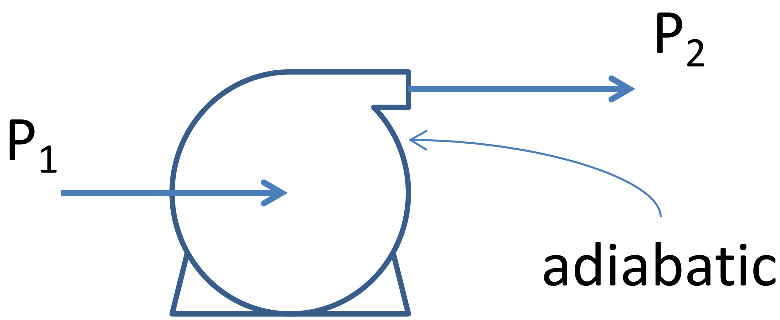
- Or $d(pv) = p dv + v dp$

$$\int_1^2 d(pv) = p_2 v_2 - p_1 v_1 = \int p dv + \int v dp$$

$$\Delta h = -w_{sh} = +\int_1^2 v dp \qquad [J / kg]$$

Example VI

- Ideal turbine or compressor



$$\Delta u + \Delta e_p + \Delta e_k = \cancel{q} - w = -\int p\,dv$$

$$\Delta u + \Delta e_p + \Delta e_k = -p_2 v_2 + p_1 v_1 + \int v\,dp$$

$$\Delta h + \Delta e_p + \Delta e_k = +\int v\,dp$$

From the previous slide

- If constant v

$$\Delta h + \Delta e_p + \Delta e_k = v\Delta p = \frac{\Delta p}{\rho}$$

v ≡ V/m

Example VII

$$CvT_2 - CvT_1 + RT_2 - RT_1 = (Cv + R)\Delta T = Cp\Delta T$$

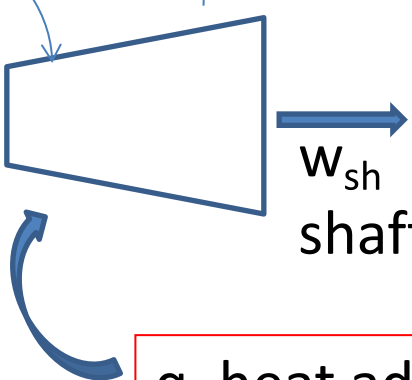
- Ideal isothermal work-producing machine

low pressure

high pressure

p_1T_1

$p_2T_2 = T_1$



w_{sh}

shaft work done

q, heat added to keep $T_2 = T_1$

Assume $\Delta e_p = \Delta e_k = 0$

Noting that $T_1 = T_2$,

then $P_1v_1 = p_2v_2$

$\Delta h = 0$

From the previous slide

$$w_{sh} = q = -\int vdp = RT \ln \frac{p_1}{p_2}$$

Or

$$\dot{W}_{sh} = \dot{Q} = \dot{n}RT \ln \frac{p_1}{p_2}$$

Example VIII

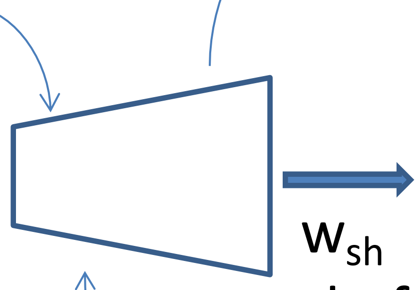
- Ideal gas frictionless adiabatic turbine or compressor

high pressure

$p_1 T_1$

low pressure

$p_2 T_2 < T_1$



w_{sh}

shaft work done

adiabatic

$$w_{flow} = w_{batch} - \Delta pv$$

$$w_{flow} = -\frac{(p_2 v_2 - p_1 v_1)}{k-1} - (p_2 v_2 - p_1 v_1)$$

$$= -\frac{k}{k-1}(p_2 v_2 - p_1 v_1) = k w_{batch}$$

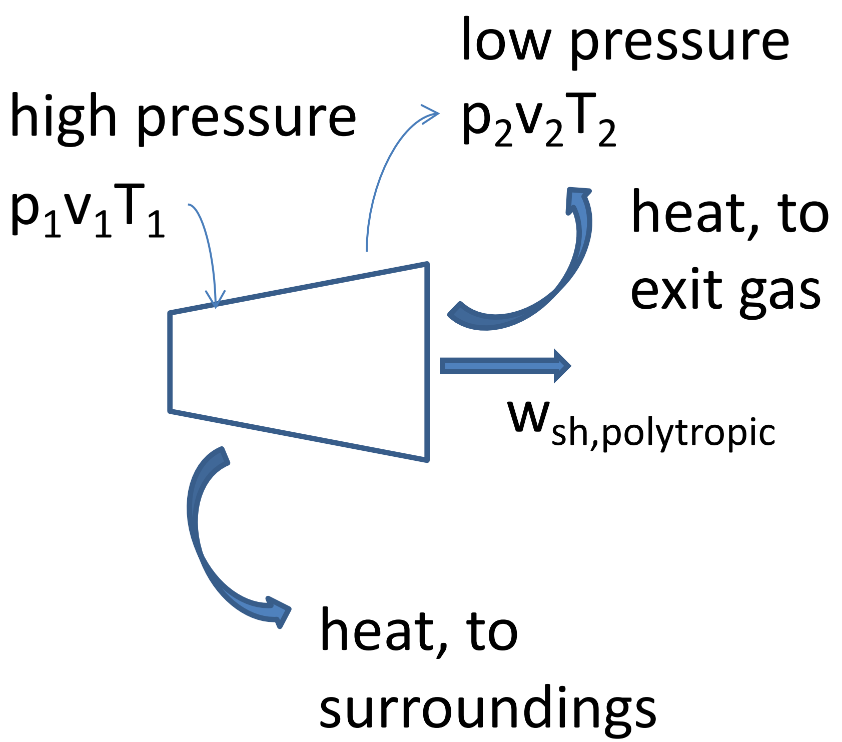
ideal gas
adiabatic reversible batch

$$\dot{W} = \frac{k p_1 \dot{V}_1}{k-1}\left[1 - \left(\frac{p_2}{p_1}\right)^{(k-1)/k}\right] = \dot{n} c_p T_1 \left[1 - \left(\frac{p_2}{p_1}\right)^{(k-1)/k}\right]$$

$$= \dot{n} c_p (T_2 - T_1) = -\frac{k}{k-1}(p_2 \dot{V}_2 - p_1 \dot{V}_1)$$

Example IX

- Real turbines and compressors



$$w_{sh,poly} = \gamma w_{batch}$$

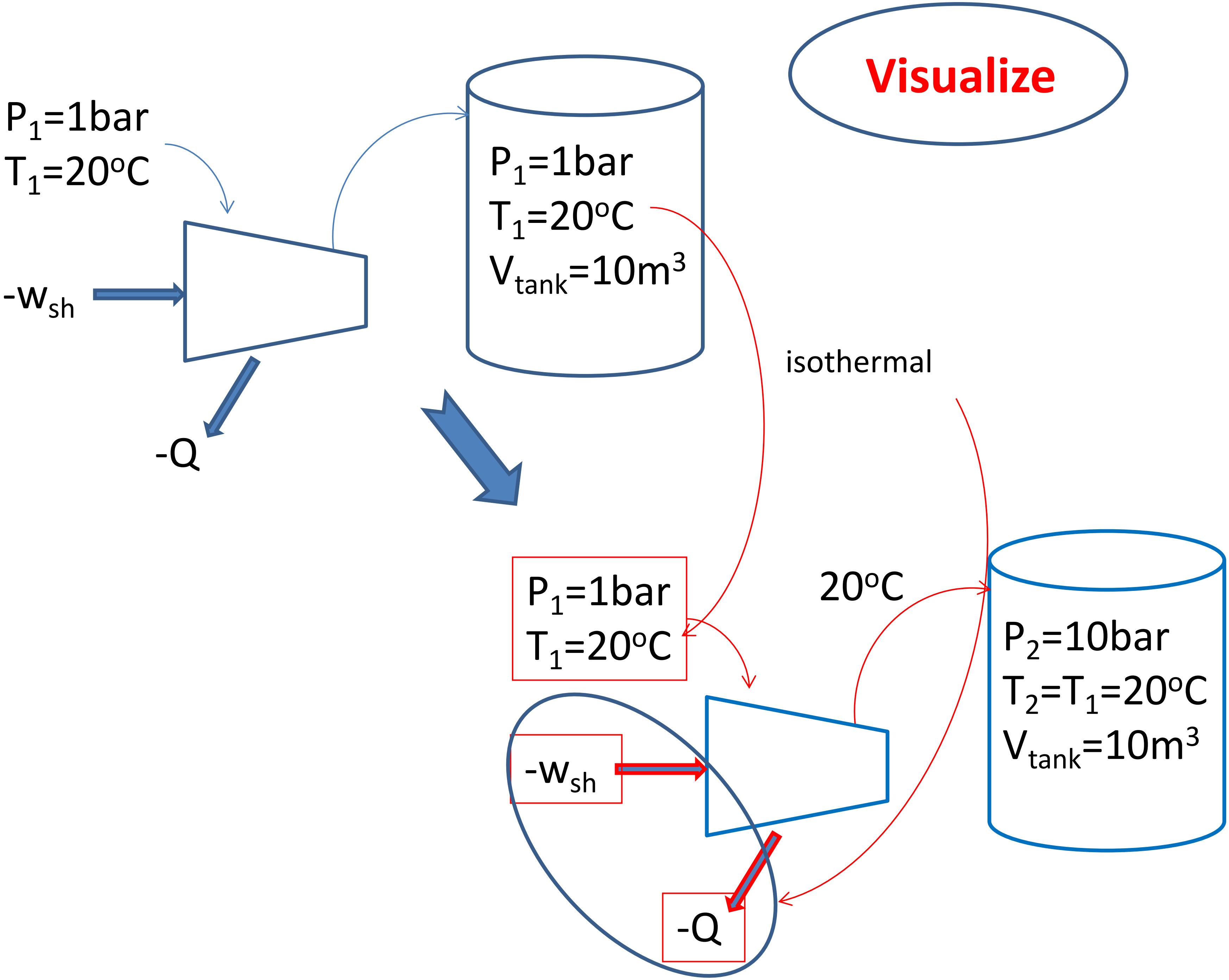
$$w_{flow} = -\frac{\left(p_2V_2 - p_1V_1\right)}{k-1} - \left(p_2V_2 - p_1V_1\right)$$

$$= -\frac{k}{k-1}\left(p_2V_2 - p_1V_1\right) = k w_{batch}$$

replace k with γ

Example X

- Pumping up a tank with an ideal gas
- A 10 m^3 tank is open to the surroundings at 20^oC and 1 bar. A compressor connected the tank pumps air into the tank. The compressor operates isothermally.
  - Find the minimum work required to pressurize the tank to 10 bar.
  - Find the heat interchange at the compressor.



Visualize
P1=1bar
T1=20oC
-wsh
-Q
P1=1bar
T1=20oC
Vtank=10m3
isothermal
P1=1bar
T1=20oC
20oC
-wsh
-Q
P2=10bar
T2=T1=20oC
Vtank=10m3

- Recall from example VII

$$w_{sh} = RT_1 \ln \frac{p_1}{p_2}$$

$$dw_{sh} = RT_1 d \ln p = RTd \ln p$$

$$w_{sh, p_1 \to p} = RT_1 \ln \frac{p_1}{p}$$

$$\boxed{dW_{sh} = w_{sh} dn = RT_1 \ln \frac{p_1}{p} dn}$$

$$\boxed{dW_{sh} = V_{\text{tank}} \ln \frac{p_1}{p} dp}$$

$$\boxed{W_{sh} = 10 m^3 \int_{10^5}^{10^6} \ln \frac{10^5}{p} dp = -13.5 \times 10^6 \,\text{J}}$$

$$m\left[\Delta h + \Delta e_p + \Delta e_k\right] = Q - W_{sh}$$

$$Q = W_{sh}$$

$$\boxed{\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots}$$

- Ideal gas EOS

$$pv = nRT$$

$$n = \frac{V_{\text{tank}} p}{RT_1}$$

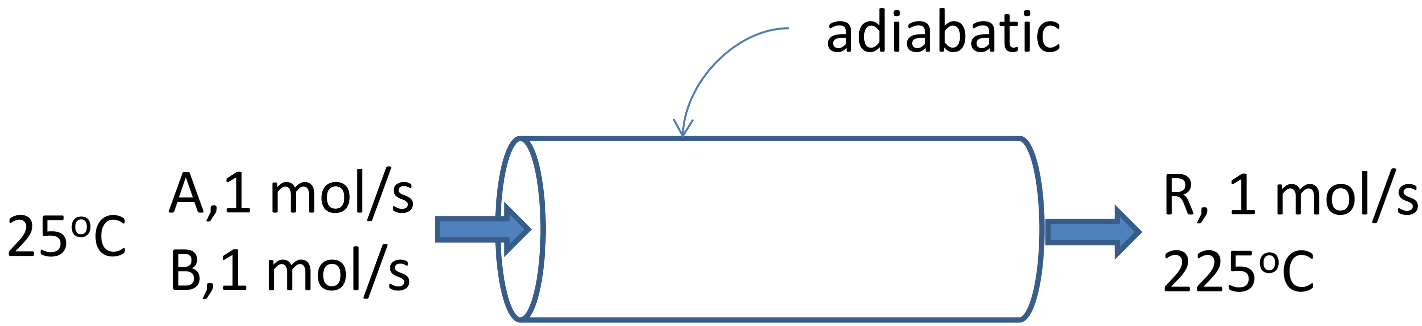
$$dn = \frac{V_{\text{tank}}}{RT_1} dp$$

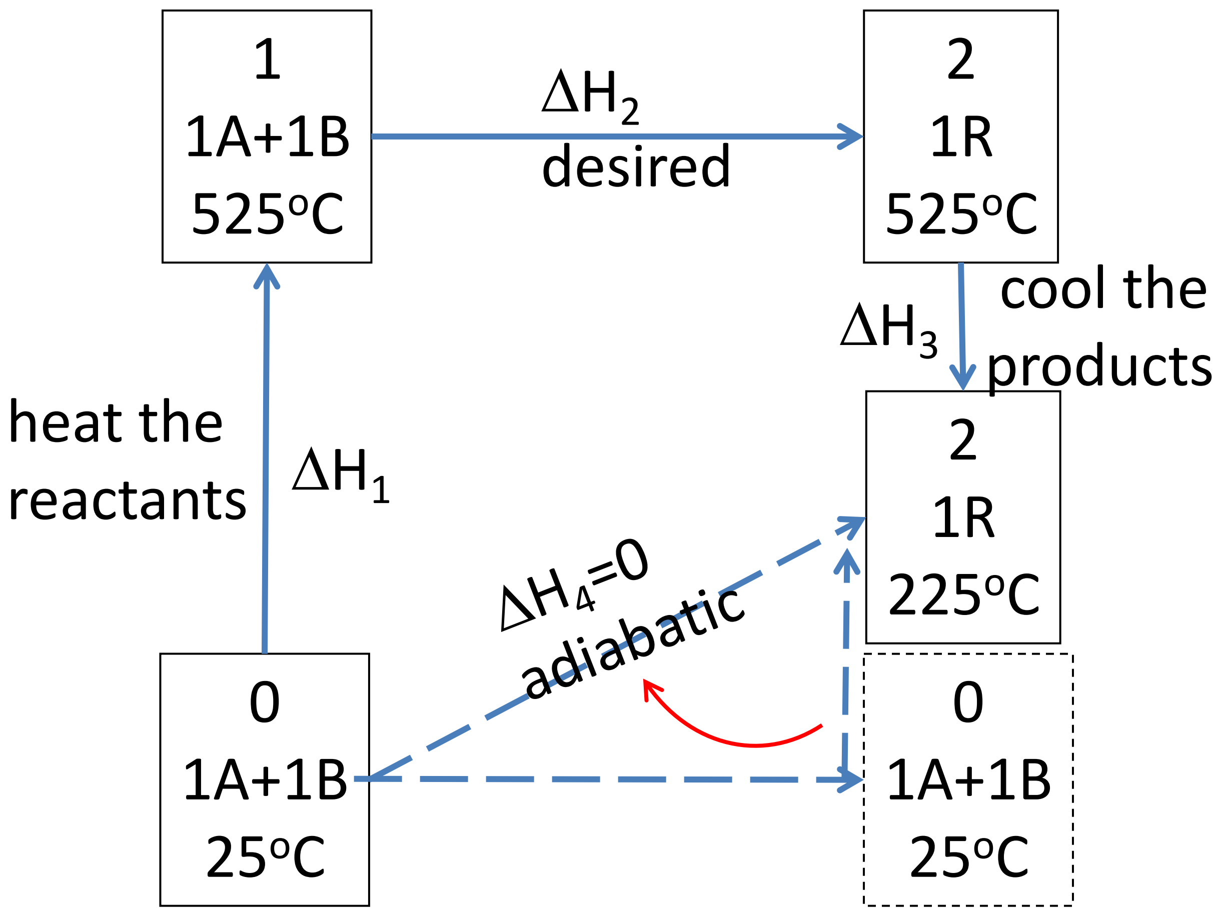
Example XI

- **The Flow Reactor** : 1 mol/s of gaseous A and 1 mol/s of gaseous B, both at 25^oC, are pumped continuously into an adiabatic mixer-reactor. They react to completion according to the stoichiometry.

$$A + B \rightarrow R$$

- The product stream, also gaseous, leaves the reactor at 225^oC. Find the ΔH_r for the above reaction at 525 ^oC.
- Data $c_{pA} = 30$, $c_{pB} = 40$, $c_{pR} = 50$ J/mol/K


adiabatic
25°C
A,1 mol/s
B,1 mol/s
R, 1 mol/s
225°C


1
1A+1B
525°C
ΔH2
desired
2
1R
525°C
cool the
products
ΔH3
heat the
reactants
ΔH1
2
1R
225°C
ΔH4=0
adiabatic
0
1A+1B
25°C
0
1A+1B
25°C

$$\Delta H + \cancelto{0}{\overline{(m\omega)} g\Delta z} + \cancelto{0}{\frac{\overline{(m\omega)}}{2}\Delta \mathbf{v}^2} = \cancelto{0}{Q} - \cancelto{0}{W_{sh}}$$

$$\Delta H_1 + \Delta H_2 + \Delta H_3 = \Delta H_4 = 0$$

$$\Delta H_2 = -\Delta H_1 - \Delta H_3$$

$$= -\left[1 \cdot c_{pA}(T_1 - T_0) + 1 \cdot c_{pB}(T_1 - T_0)\right] - 1 \cdot c_{pR}(T_3 - T_2)$$

$$= -20 \text{ kJ/mol}$$

Double Interpolation

- Find s of water at v = 0.25 m^3/kg, h = 3100 kJ/kg.
- This is a superheated state: at $v_g \approx 0.25$ m^3/kg we see that $h_g \approx 2700$ kJ/kg; the water at our actual volume would have a higher energy than at saturation and the state is superheated.
- 1. go to the superheated tables, looking for the numbers for v = 0.25 m^3/kg, and find the P region where h is close to 3100. At 1000 kPa h will be a little low, and at 1200 it will be a little high. The four data points are:

1. Find the raw data

P = 1000			P = 1200		
v	h	s	v	h	s
0.2327	2943	6.925	0.2345	3154	7.212
0.2579	3051	7.123	0.2548	3261	7.377

2. Interpolate holding P constant and using v = 0.25 m^3/kg:

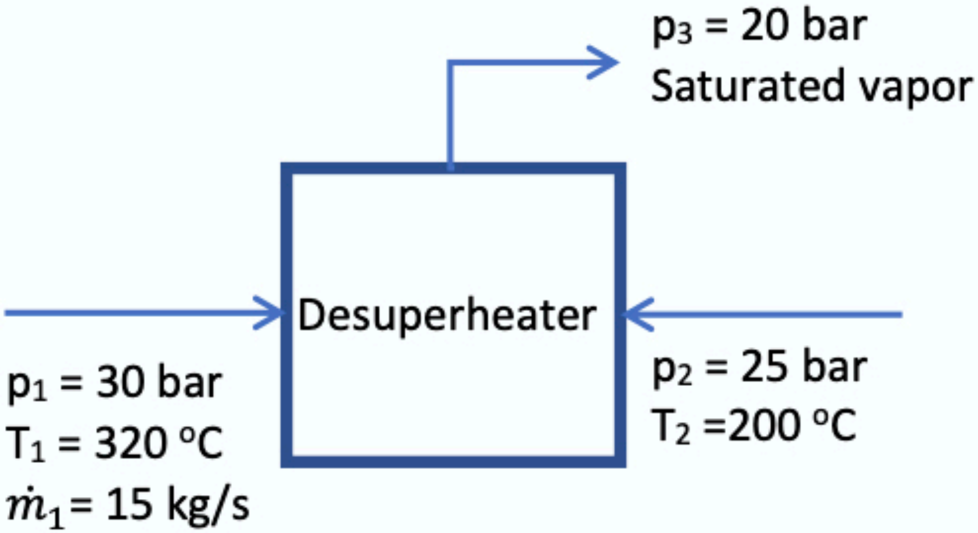
P = 1000			P = 1200		
v	h	s	v	h	s
0.25	3017	7.061	0.25	3236	7.338

3. interpolate using h = 3100 kJ/kg:

P = 1076		
v	h	s
0.25	3100	7.166

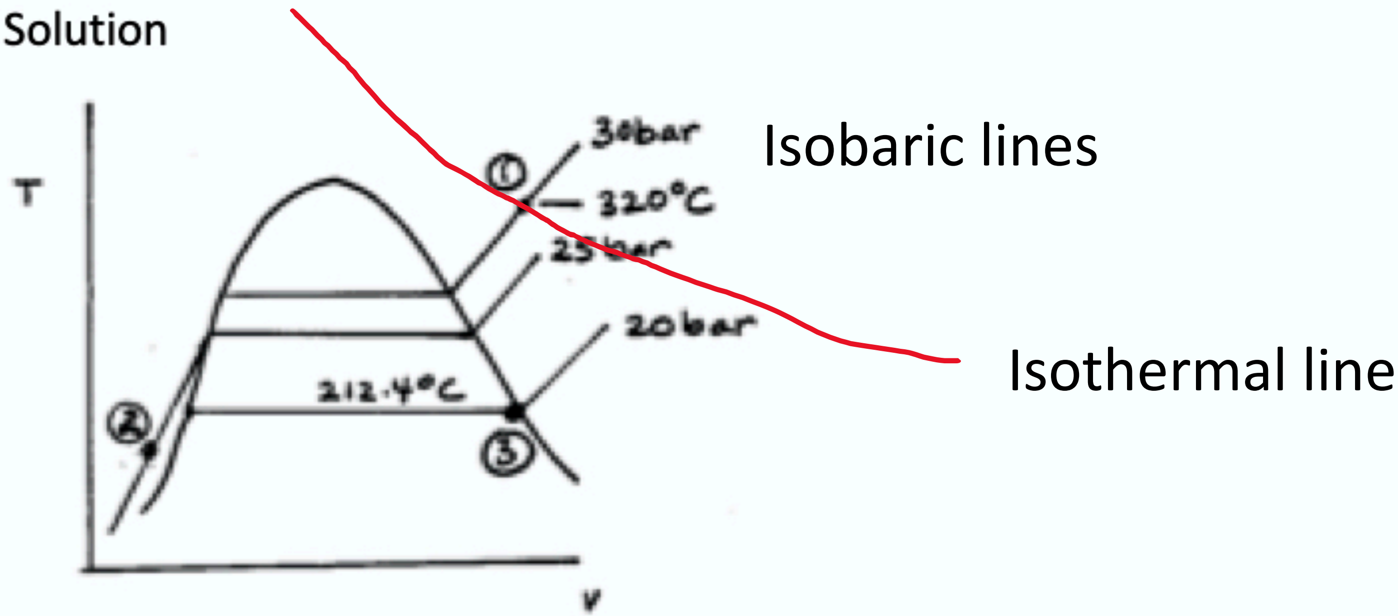
Quiz VI control volume

As shown in the Figure 15 kg/s of steam enters a desuperheater operating at steady state at 30 bar, 320°C, where it is mixed with liquid water at 25 bar and temperature T_2 to produce saturated vapor at 20 bar. Heat transfer between the device and its surroundings and kinetic and potential energy effects can be neglected. If T_2 = 200°C, determine the mass flow rate of liquid, $\dot{m}_2$, in kg/s.



where h_1(30 bar, 320°C) = 3043.4 [J/kg]; h_2 (25 bar, 200°C) = 852.8 [J/kg]; h_3(20 bar, ? °C) = 2799.5 [J/kg]

Solution



The mass rate balance at steady state is

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

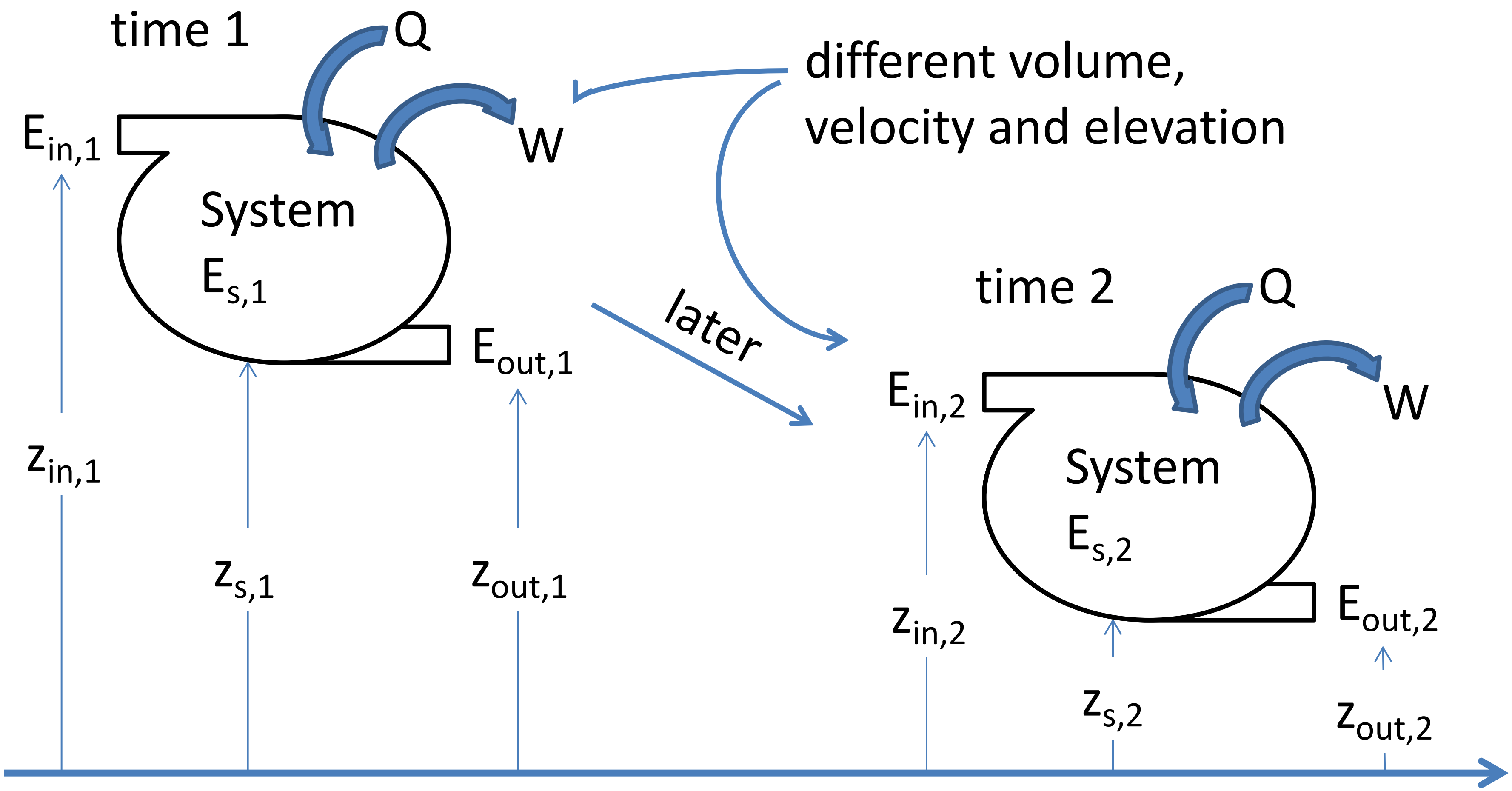
The energy rate balance at stead state is

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 = (\dot{m}_1 + \dot{m}_2) h_3$$

$$\dot{m}_2 = \dot{m}_1 \left(\frac{h_3 - h_1}{h_2 - h_3}\right) = \left(15\,\frac{kg}{s}\right)\left(\frac{2799.5 - 3043.4}{852.8 - 2799.5}\right) = 1.88 kg/s$$

UNSTEADY STATE FLOW SYSTEMS

Unsteady state flow systems



$$\Delta \mathbf{E}_{system} = \text{(all energy inputs)} - \text{(all energy outputs)}$$

$$= - \Delta \mathbf{E}_{streams} + Q - W$$

$$\underbrace{m_2(u+e_p+e_k)_2 - m_1(u+e_p+e_k)_1}_{\textit{system}} + \underbrace{m_{out}(u+e_p+e_k)_{out} - m_{in}(u+e_p+e_k)_{in}}_{\textit{streams}}$$

$$= Q - W$$

where $W = W_{sh} + W_{pv,system} + W_{pv,streams}$

$$m_2(u+e_p+e_k)_2 - m_1(u+e_p+e_k)_1 + m_{out}(h+e_p+e_k)_{out} - m_{in}(h+e_p+e_k)_{in}$$

$$= Q - (W_{sh} + W_{pv,system})$$

$$= Q - \left(W_{sh} + \int_{V_1}^{V_2} p_{system} dv \right)$$

volume change in system

Example I

- Filling a glass with water

Hot water (80^oC) from a kettle is poured into a completely insulated styrofoam cup. Apply the general equation to the cup to find the temperature of the water in the cup.

$$(m_{cup}u_{cup})_2 + (m_{water}u_{water})_2 - (m_{cup}u_{cup})_1 - \overset{\boxed{\text{water 1}}}{m_{kettle}h_{kettle}} = Q - p_2v_2$$

assume $u_{cup1} = u_{cup2}$, $Q=0$, $p_2 = 1$ bar, v_2 = hot water

$$\cancel{(m_{cup}u_{cup})_2} + (m_{water}u_{water})_2 - \cancel{(m_{cup}u_{cup})_1} - m_{kettle}h_{kettle} = \cancel{Q} - p_2v_2$$

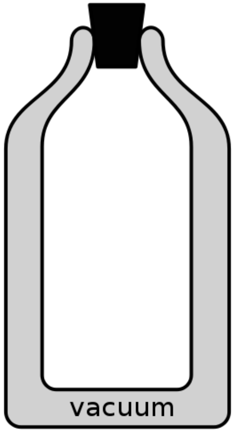
$$u_{water2} - h_{kettle} = -\, p_2v_{water2} \quad \text{or} \quad h_{water2} = h_{kettle}\,,\ T_{water2} = T_{kettle} = 80^oC$$

$$\Delta h_{water} = 0, \quad Cp\Delta T_{water} = 0$$

Example II

- Filling an evacuated tank with an ideal gas

A valve on an evacuated insulated tank is opened. Air (an ideal gas) rushes in and the pressure equalizes. The valve is then quickly closed. What is the temperature of the gas in the tank if room temperature is 27^{o}C and pressure is 1 bar.



$$m_2(u+\cancel{e_p}+\cancel{e_k})_2 - \cancel{m_1}(u+e_p+e_k)_1 + \cancel{m_{out}}(h+e_p+e_k)_{out} - m_{in}(h+\cancel{e_p}+\cancel{e_k})_{in}$$

$$= Q - (W_{sh} + \cancel{W_{pv,system}})$$

$$m_2u_2 - m_{in}h_{in} = 0 \qquad u_2 = h_{in} \quad \text{or} \quad c_vT_2 = c_pT_{in}$$

$= m_2$,means stream in becomes part of system

$$T_2 = \left(\frac{29.1}{29.1 - 8.314} \right)(300) = 420\text{K} = 147^{\circ}\text{C}$$

An extension of the previous example has some fluid originally in the tank.

$$m_2(u+e_p+e_k)_2 - m_1(u+e_p+e_k)_1 + m_{out}(h+e_p+e_k)_{out} - m_{in}(h+e_p+e_k)_{in}$$
$$= Q - (W_{sh} + W_{pv,system})$$

$$m_2u_2 - m_1u_1 - (m_2 - m_1)\, h_{in} = 0$$

Since m_2 and u_2 are unknown, one may have to use trial and error to solve.

Example IV

- A large unused exhibition hall (50 m x 40 m x 10 m) is to be prepared for a show and has to be heated from 0^oC to 25^oC. How many 1.5 kW portable heaters operating for 24 hrs would be needed for this job?
- Assume the pressure stays at 1 bar, air leaks out of the hall. Only account for the heating of air not walls, fixtures and furniture.

$$n_2(u\cancel{+e_p+e_k})_2 - n_1(u\cancel{+e_p+e_k})_1 + n_{out}(h\cancel{+e_p+e_k})_{out} - \cancel{n_{in}}(h+e_p+e_k)_{in}$$
$$= Q - (\cancel{W_{sh}} + \cancel{W_{pv,system}})$$

wall is rigid

$$n_2u_2 - n_1u_1 + \int^{n_1-n_2} h_{out}dn = Q$$

Part of the system becomes stream

$$n_2c_vT_2 - n_1c_vT_1 + \int^{n_1-n_2} c_pTdn = Q$$

changes as n changes

$$pv = nRT, \text{ or } nT = \frac{pv}{R} = \text{const.} = n_1T_1$$

$$dn_{\substack{in\\vessel}} = \frac{n_1T_1}{-T^2}dT = -dn_{out}$$

$$\cancel{n_2c_vT_2\left(\frac{n_1T_1}{n_2T_2}\right)} - n_1c_vT_1 + c_p\int_{T_2}^{T_1} T\left(\frac{n_1T_1}{-T^2}\right)dT = Q$$

$$Q = \frac{c_p pV}{R}\ln\frac{T_2}{T_1} = \frac{29.1(100000)(20000)}{8.314}\ln\frac{298}{273}$$

$$= 613\times10^6 \text{ J/day}. \quad \boxed{\frac{613\times10^6\text{ J/day}}{24\times3600\text{ s/day}}\left(\frac{\text{heater}}{1500\text{W}}\right)\approx 5}$$