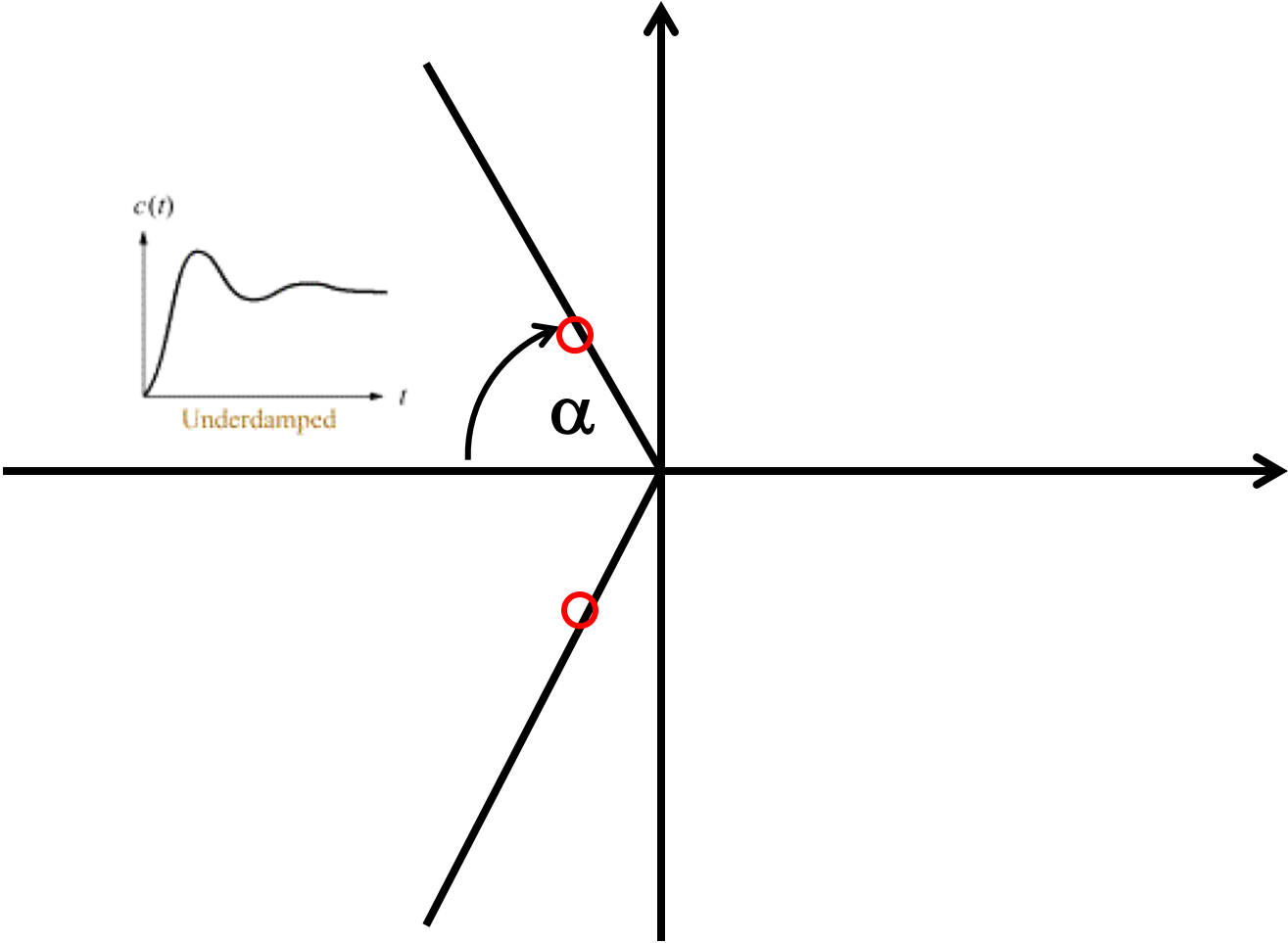


# **Transient Response**

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# Stability



- For a 2<sup>nd</sup> order system

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

- Then the transient response is

$$c(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1 - \zeta^2}} \sin\left(\omega_n \sqrt{1 - \zeta^2} t + \alpha\right)$$

- where

$$\alpha = \cos^{-1}(\zeta)$$

- We have

$$t_p = \frac{\pi}{\omega_n \sqrt{1 - \zeta^2}}$$

$$\%OS = \exp\left(\frac{-\zeta\pi}{\sqrt{1 - \zeta^2}}\right)$$

# The Dominant Poles

- Transient performance is often dominated by the pair of complex conjugate poles located closest to the origin. This occurs when the other poles are far to the left of the dominant poles, or other poles are near a zero. The pair is conventionally referred to as the dominant poles.
- A general guideline, the ratios of the real parts are at least five, and there are no zeros nearby.

- For **unity-feedback** systems,

$$[t_p]_{modified} = \frac{1}{\sqrt{1 - \zeta^2} \omega_n} \left( \pi - \sum \phi_z + \sum \phi_p \right)$$

- $\sum \phi_z$  = sum of the angles from the zeros of C/R to one of the dominant poles
- $\sum \phi_p$  = sum of the angles from the zeros of C/R to one of the dominant poles

- The maximum percent overshoot

$$\begin{aligned} \%OS &= \left[ \left( \frac{P_1}{|P_1 - P_0|} \right) \left( \frac{P_2}{|P_2 - P_0|} \right) \dots \right] \left[ \left( \frac{|Z_1 - P_0|}{Z_1} \right) \left( \frac{|Z_2 - P_0|}{Z_2} \right) \dots \right] e^{-\zeta \omega_n t_p} 100\% \end{aligned}$$

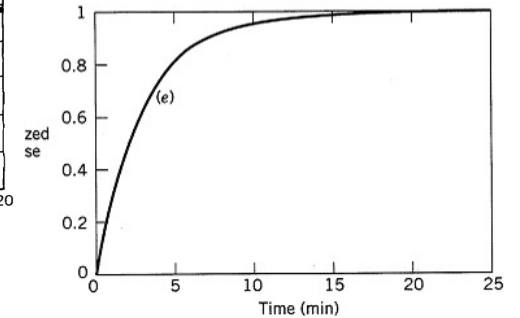
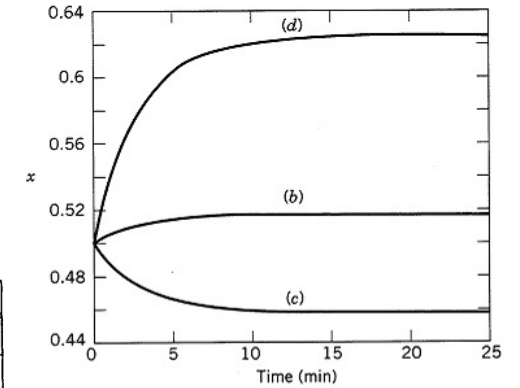
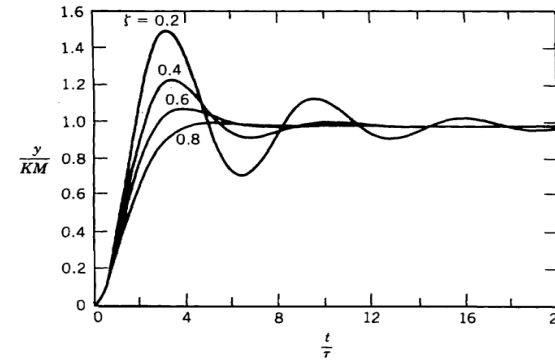
$$\bullet \left[ \left( \frac{P_1}{|P_1 - P_0|} \right) \left( \frac{P_2}{|P_2 - P_0|} \right) \cdots \right] = \frac{\text{product of distances from all poles of } C/R \text{ to origin excluding distances of two dominant poles from origin}}{\text{product of distances from all other poles of } C/R \text{ to dominant pole } P_0 \text{ excluding distance between dominant poles}}$$

$$\bullet \left[ \left( \frac{|Z_1 - P_0|}{Z_1} \right) \left( \frac{|Z_2 - P_0|}{Z_2} \right) \cdots \right] = \frac{\text{product of distances from all zeros of } C/R \text{ to dominant pole } P_0}{\text{product of distances from all zeros of } C/R \text{ to origin}}$$

$$(G(s)H(s)) \rightarrow \frac{E}{R} = \frac{C}{R} = \frac{G(s)H(s)}{1 + G(s)H(s)}$$

$$G(s) \rightarrow \frac{C}{R} = \frac{G(s)}{1 + G(s)H(s)}$$

$$G(s) \rightarrow \frac{E}{R} = \frac{R - CH}{R} = 1 - \frac{G(s)H(s)}{1 + G(s)H(s)} = \frac{1}{1 + G(s)H(s)}$$



Equivalent

# Example

$$G_c(s)G(s)H(s) = \left( \frac{s + 0.2779}{s + 0.01} \right) \frac{22.19}{s(s + 4)(s + 5)}$$

$$\begin{aligned} \frac{C}{R} &= \frac{G_c(s)G(s)H(s)}{1 + G_c(s)G(s)H(s)} = \frac{\left( \frac{s + 0.2779}{s + 0.01} \right) \frac{22.19}{s(s + 4)(s + 5)}}{1 + \left( \frac{s + 0.2779}{s + 0.01} \right) \frac{22.19}{s(s + 4)(s + 5)}} \\ &= \frac{22.19(s + 0.2779)}{s(s + 0.01)(s + 4)(s + 5) + 22.19(s + 0.2779)} \\ &= \frac{s^4 + 9.01s^3 + 20.09s^2 + 22.39s + 6.1666}{22.19(s + 0.2779)} \\ &= \frac{(s + 6.3911)(s + 0.388)(s + 1.1154 \pm j1.1147)}{22.19(s + 0.2779)} \end{aligned}$$



- The system has one zero ( $s + 0.2779$ ), two poles other than the dominants,  $(s + 6.3911)(s + 0.388)$  plus the conjugate

- $\phi_{z_1} = \arctan \frac{1.115}{+0.2779-1.115} = 126.9 \times 0.01745329 = 2.21$

- $\sum \phi_z = 2.21$

- $\phi_{P_1} = \arctan \frac{1.115}{0.388-1.115} = 123.1$

- $\phi_{P_2} = \arctan \frac{1.115}{6.3911-1.115} = 11.9$

- $\sum \phi_p = 123.1 + 11.9 = 135 \times 0.01745329 = 2.36$

$$[t_p]_{modified} = \frac{1}{\sqrt{1 - \zeta^2} \omega_n} \left( \pi - \sum \phi_z + \sum \phi_p \right)$$

$$= \frac{1}{1.115} (\pi - 2.21 + 2.36) = 2.95 \text{ sec}$$

- Dominant poles:  $s + 1.115 \pm j1.115$ ,
- Use general form,  $\zeta = \cos \alpha = 0.707$ ,  $\omega_n = 1.5769$
- If one simply assume that the transient response is governed by the pair of complex-conjugate poles,  $s + 1.115 \pm j1.115$ , then

$$t_p = \frac{\pi}{\omega_n \sqrt{1 - \zeta^2}} = \frac{3.14}{1.115} = 2.82 \text{ sec}$$

$$\%OS = \exp\left(\frac{-\zeta\pi}{\sqrt{1 - \zeta^2}}\right) = \exp\left(\frac{-0.707\pi}{\sqrt{0.5}}\right) = 4.32\%$$

$$s^2+2.4s+2.88 \quad s^3+9s^2+18.72s+19.008$$

- Uncompensated system

- $G(s)H(s) = \frac{K}{s(s+4)(s+5)}$

- $\frac{C}{R} = \frac{G(s)H(s)}{1+G(s)H(s)} = \frac{\frac{K}{s(s+4)(s+5)}}{1+\frac{K}{s(s+4)(s+5)}} = \frac{K}{s(s+4)(s+5)+K} = \frac{K}{s^3+9s^2+20s+K} = \frac{K}{s^3+9s^2+18.72s+19.008} = \frac{22.19}{(s+6.6)(s+1.2 \pm j1.2)}$

- Dominant poles:  $s + 1.2 \pm j1.2$ ,
- Use general form,  $\zeta = \cos \alpha = 0.707$ ,  $\omega_n = 1.697$
- $\sum \phi_z = 0$
- $\phi_{P_1} = \arctan \frac{1.2}{6.6-1.2} = 12.5$

$$\sum \phi_p = 12.5 = 12.5 \times 0.01745329 = 0.2182$$

$$[t_p]_{modified} = \frac{1}{\sqrt{1-\zeta^2}\omega_n} \left( \pi - \sum \phi_z + \sum \phi_p \right) = \frac{1}{1.2} (\pi - 0 + 0.218) = 2.8 \text{ sec}$$

- Dominant poles:  $s + 1.115 \pm j1.115$ ,
- Use general form,  $\zeta = \cos \alpha = 0.707$ ,  $\omega_n = 1.5769$
- If one simply assume that the transient response is governed by the pair of complex-conjugate poles,  $s + 1.115 \pm j1.115$ , then

$$t_p = \frac{\pi}{\omega_n \sqrt{1 - \zeta^2}} = \frac{3.14}{1.2} = 2.62 \text{ sec}$$

$$\%OS = \exp\left(\frac{-\zeta\pi}{\sqrt{1 - \zeta^2}}\right) = \exp\left(\frac{-0.707\pi}{\sqrt{0.5}}\right) = 4.32\%$$

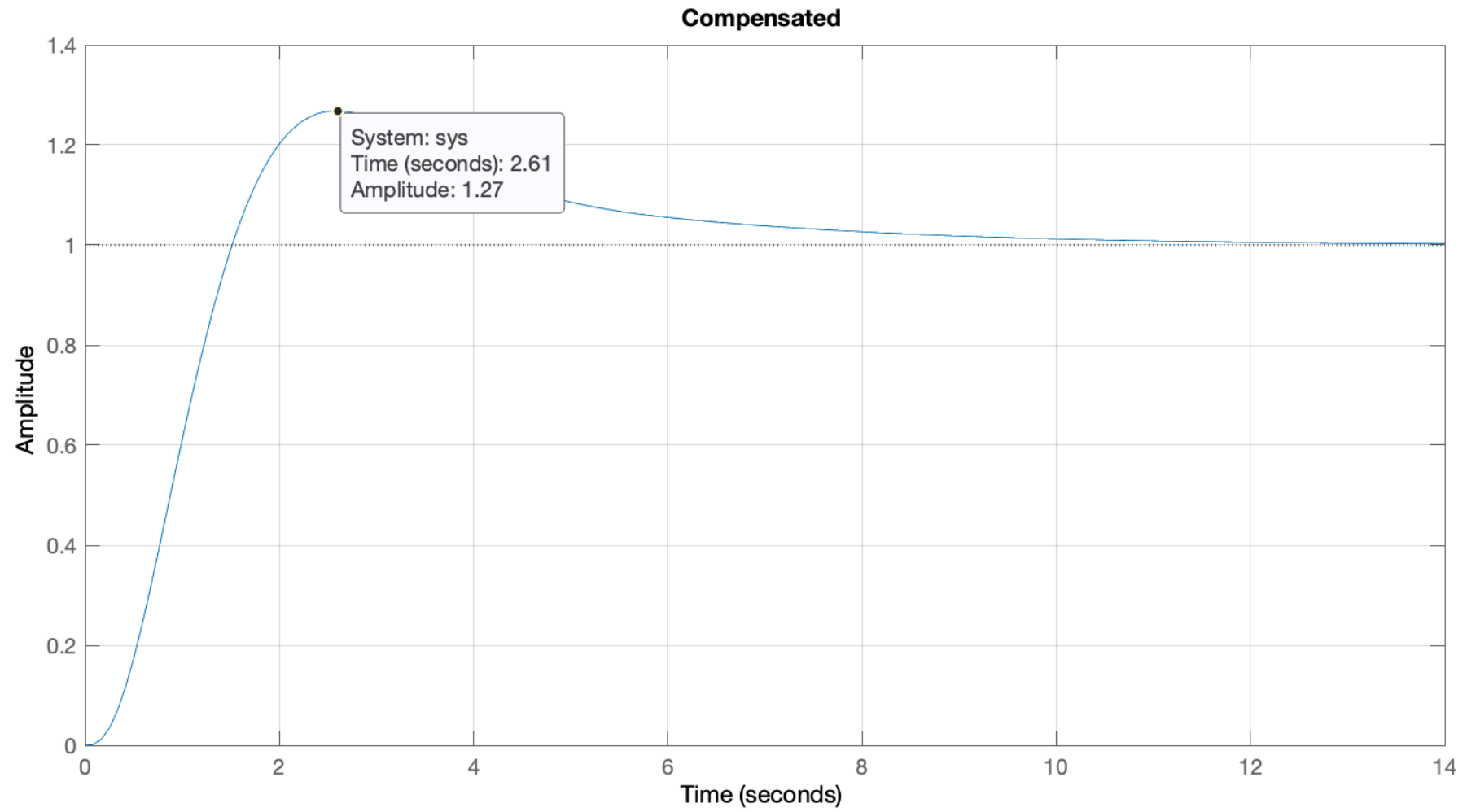
- We turn to percentage overshoot %OS
- one zero  $(s + 0.2779)$ , two poles  $(s + 6.3911)(s + 0.388)$
- $|P_1 - P_0| = \sqrt{(-0.388 + 1.115)^2 - (j0 - j1.115)^2} = 1.3311$
- $|P_2 - P_0| = \sqrt{(-6.911 + 1.115)^2 - (j0 - j1.115)^2} = 5.9023$
- $|Z_1 - P_0| = \sqrt{(-0.2779 + 1.115)^2 - (j0 - j1.115)^2} = 1.3943$

$$\begin{aligned}
 \%OS &= \left( \frac{P_1}{|P_1 - P_0|} \right) \left( \frac{P_2}{|P_2 - P_0|} \right) \frac{|Z_1 - P_0|}{Z_1} e^{-\zeta \omega_n t_p} \\
 &= \left( \frac{0.388}{1.3311} \right) \left( \frac{6.3911}{5.9023} \right) \frac{1.3943}{0.2779} e^{-0.707 \cdot 1.5769 \cdot 2.95} = 1.608 e^{-3.28886} \\
 &= 5.997\%
 \end{aligned}$$

- Uncompensated percentage overshoot %OS
- no zero, one poles  $(s + 6.6)(s + 1.2 \pm j1.2)$
- $|P_1 - P_0| = \sqrt{(-6.6 + 1.2)^2 - (j0 - j1.2)^2} = 5.5317$

$$\begin{aligned} \%OS &= \left( \frac{P_1}{|P_1 - P_0|} \right) e^{-\zeta \omega_n t_p} = \left( \frac{6.6}{5.5317} \right) e^{-0.707 \cdot 1.697 \cdot 2.8} \\ &= 1.1931 e^{-3.28886} = 4.15\% \end{aligned}$$

step(num,den)  
grid



step(num,den)  
grid

