Heat Engines and the Carnot Cycle

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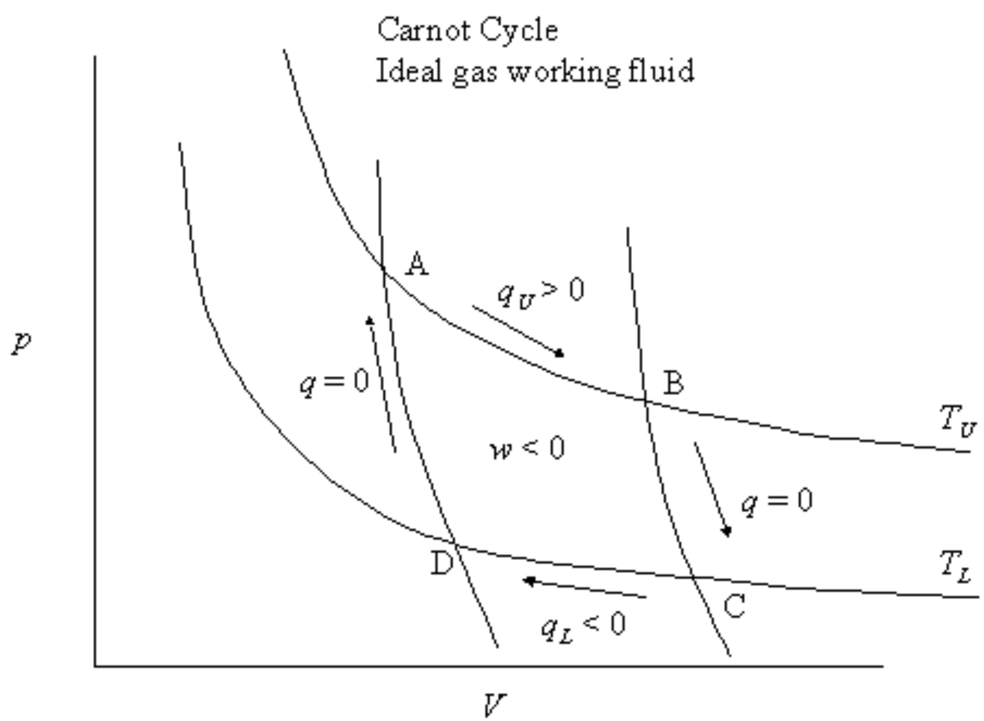
Heat Engine and Cyclic Process

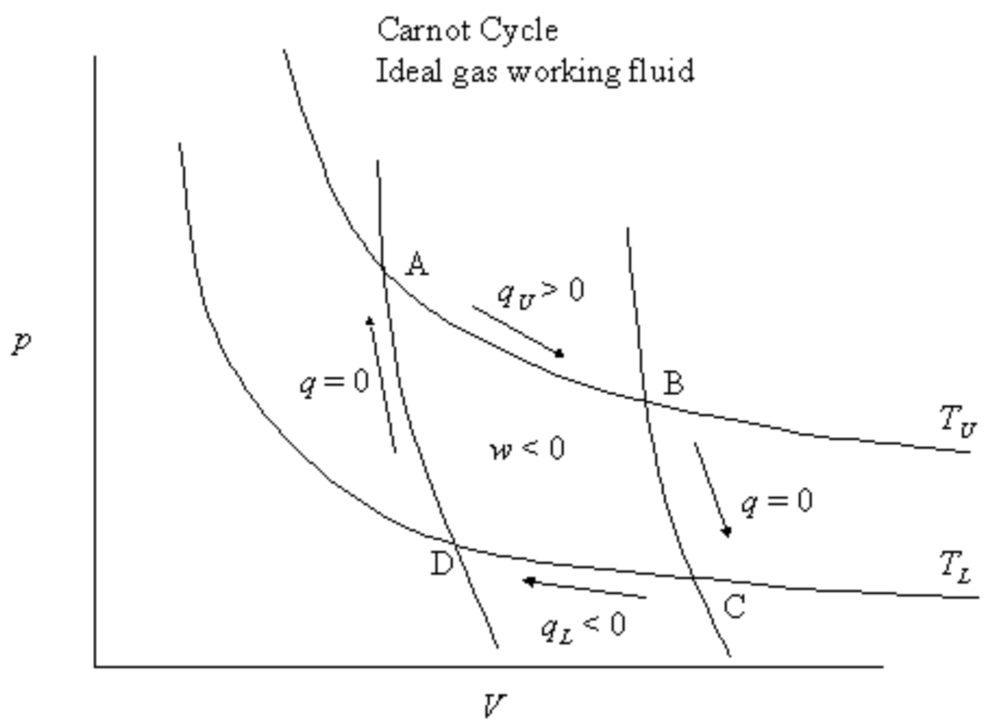
A heat engine acts by transferring energy from a warm region to a cool region of space and, in the process, converting some of that thermal energy to mechanical work. "Cyclic" means that the system returns to its initial state at the end of each cycle so that there is no permanent change in the system.

Carnot Cycle



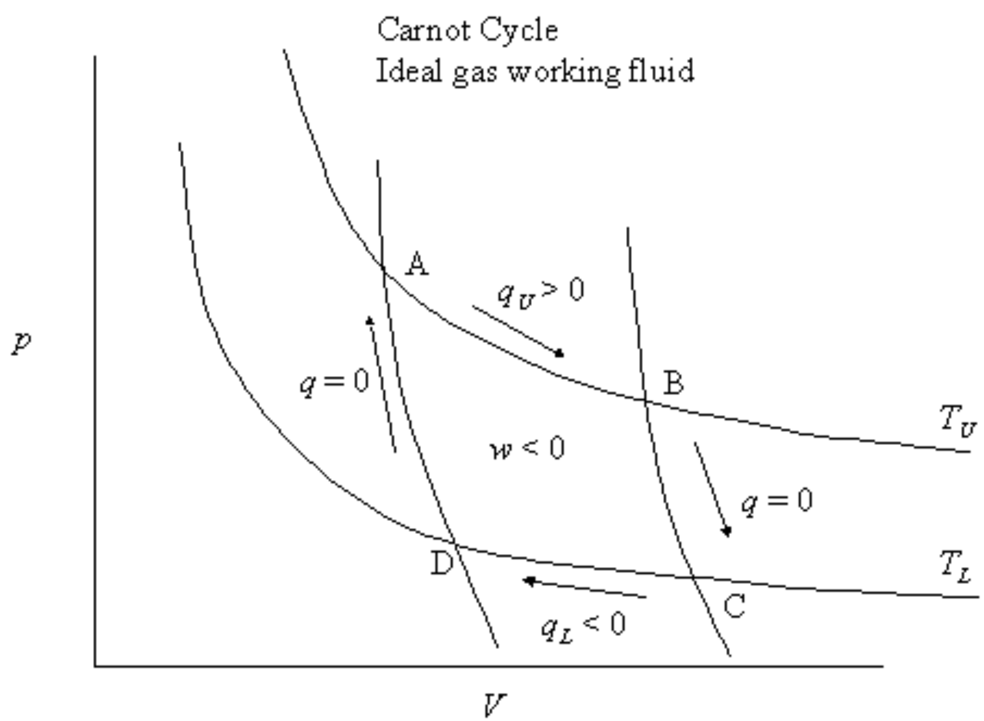
The curves labeled T_U *and* T_L are isotherms. T_U is the temperature of the upper temperature heat bath (reservoir) and T_L is the temperature of the lower temperature heat bath. The two steep curves, BC and DA, are adiabatic curves.

Carnot Cycle



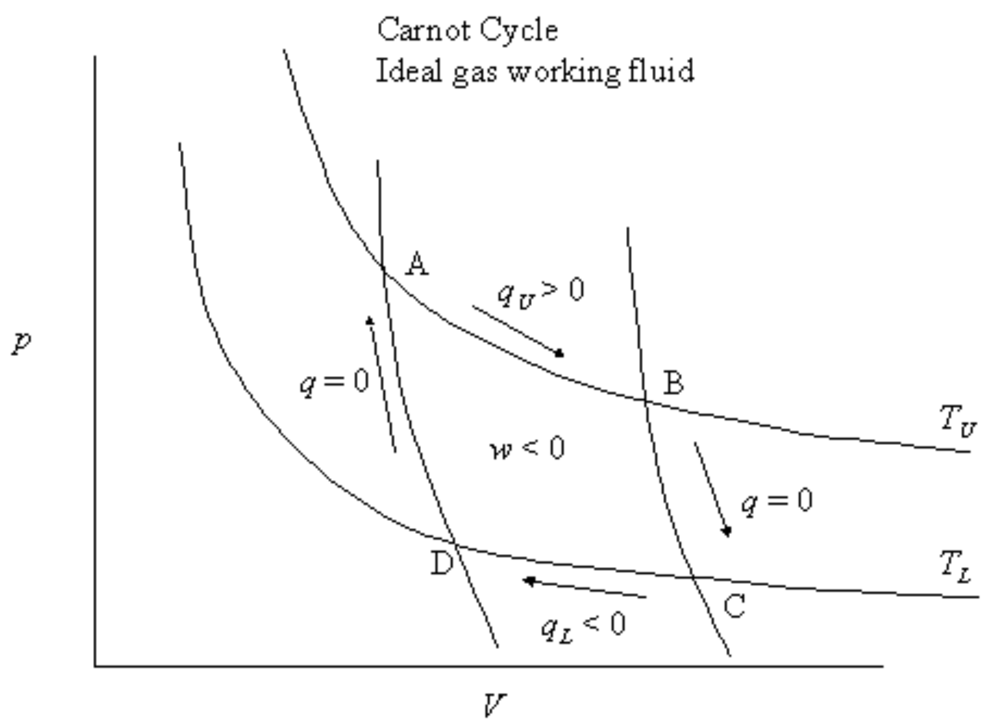
From point A, the system undergoes an isothermal expansion at temperature, T_U, to point B. In this isothermal expansion the system absorbs heat from the upper heat bath ($q_U > 0$) and does work on the surroundings (*w* is defined as work done on the system so $w_{AB} < 0$).

Carnot Cycle



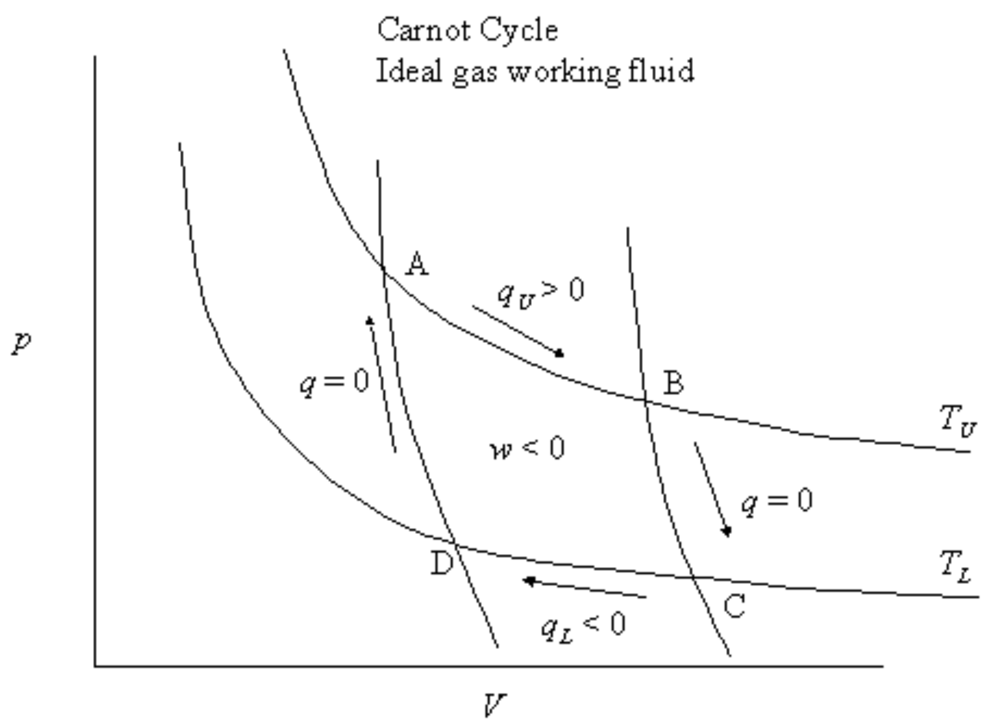
From point B, the system is then isolated from the heat bath and is expanded adiabatically to the point C. There is no heat in this adiabatic expansion, but the work for this step is also negative ($w_{BC} < 0$).

Carnot Cycle



- **At point C the system is placed in contact with a heat bath at T_L and undergoes an isothermal compression to point D. For this segment of the cycle $q_L < 0$ and $w_{CD} > 0$ because the surroundings are now doing work on the system and heat is being dissipated to the heat bath at T_L**

Carnot Cycle



At point D the system is again isolated from the heat baths and compressed adiabatically to point A. In this adiabatic compression the heat, of course, is zero and the work is positive ($w_{DA} > 0$)

The system completes a cycle.

Carnot Cycle

- **The Carnot cycle is a particular thermodynamic cycle proposed by Nicolas Léonard Sadi Carnot in 1824 and expanded by Benoit Paul Émile Clapeyron in the 1830s and 40s.**
- **What makes the Carnot cycle special, is that it is the most efficient existing cycle converting a given amount of thermal energy into work.**

The Efficiency

- **The heat, *q*, for the whole cycle is**

$$q = q_{AB} + q_{CD}$$

- **the work for the entire cycle is,**

$$w = w_{AB} + w_{BC} + w_{CD} + w_{DA}$$

- **Since the initial state of the cycle is the same as the final state, the change in *U*, the internal energy, is zero.**

$$\Delta U = q + w = 0$$

The Efficiency

- **Therefore,**

$$q = -w$$

- **The Carnot cycle is operating as a heat engine, *w* < 0, so that -*w* > 0 and *q* > 0.**
- **As a heat engine, heat is absorbed and converted into work.**
- **the efficiency, η, of the cycle is**

$$\eta = \frac{-w}{q_{AB}}$$

The Efficiency

- **Therefore,**

$$\eta = \frac{q_{AB} + q_{CD}}{q_{AB}}$$

- **the A→ B segment of the cycle is at the temperature of the upper heat bath, *T_U, and the segment C→ D is at the temperature of the lower heat bath, T_L. Thus***

$$\eta = \frac{q_U + q_L}{q_U} = 1 + \frac{q_L}{q_U}$$

The Efficiency

- **For isothermal expansions and compressions of ideal gases,**
- $\Delta U = q + w = 0$**, therefore,**

$$q_U = -w_{AB} = nRT_U \ln \frac{V_B}{V_A}$$

$$q_L = -w_{CD} = nRT_L \ln \frac{V_D}{V_C}$$

- **where**

$$w_{AB} = -\int_{V_A}^{V_B} p dV = -\int_{V_A}^{V_B} \frac{nRT}{V} dV = -nRT \ln \frac{V_B}{V_A}$$

The Efficiency

- **Therefore,**

$$\eta = 1 + \frac{nRT_L \ln \frac{V_D}{V_C}}{nRT_U \ln \frac{V_B}{V_A}} = 1 + \frac{T_L \ln \frac{V_D}{V_C}}{T_U \ln \frac{V_B}{V_A}}$$

Adiabatic Processes of Ideal Gas

- **An adiabatic process is that heat transfer to the system is zero,** $\delta q = 0$
- **From the first law and ideal gas,**

$$dU + \delta w = \delta q = 0$$

$$\delta w = pdV$$

- **From the Joule experiment,**

$$dU = n\,C_v\,dT \text{ and } C_v = \alpha R \quad (\textbf{kinetic theory})$$

$$dU = \alpha nRdT = \alpha d(PV) = \alpha\,(pdV + Vdp)$$

Adiabatic Processes of Ideal Gas

$$-pdV = \alpha pdV + \alpha Vdp$$

$$-(\alpha+1)pdV = \alpha Vdp$$

$$-(\alpha+1)\frac{dV}{V} = \alpha\frac{dp}{p}$$

$$\ln\left(\frac{p}{p_0}\right) = -\frac{\alpha+1}{\alpha}\ln\left(\frac{V}{V_0}\right)$$

$$\left(\frac{p}{p_0}\right)\left(\frac{V}{V_0}\right)^{\frac{\alpha+1}{\alpha}} = 1$$

$$pV^{\frac{\alpha+1}{\alpha}} = p_0 V_0^{\frac{\alpha+1}{\alpha}} = pV^{\gamma} = \text{const.}$$

The Efficiency

- **For adiabatic paths B→ C and D→ A,**

$$p_B V_B^\gamma = p_C V_C^\gamma$$

$$p_D V_D^\gamma = p_A V_A^\gamma$$

$$\frac{nRT_U}{V_B} V_B^\gamma = \frac{nRT_L}{V_C} V_C^\gamma$$

$$T_U V_B^{\gamma-1} = T_L V_C^{\gamma-1}$$

$$\frac{T_U}{T_L} = \frac{V_C^{\gamma-1}}{V_B^{\gamma-1}} \quad \text{and} \quad \frac{T_U}{T_L} = \frac{V_D^{\gamma-1}}{V_A^{\gamma-1}}$$

$$\frac{V_C^{\gamma-1}}{V_B^{\gamma-1}} = \frac{V_D^{\gamma-1}}{V_A^{\gamma-1}}$$

$$\frac{V_C}{V_B} = \frac{V_D}{V_A}$$

The Efficiency

- **Therefore,**

$$\eta = 1 + \frac{nRT_L \ln \frac{V_D}{V_C}}{nRT_U \ln \frac{V_B}{V_A}} = 1 + \frac{T_L \ln \frac{V_A}{V_B}}{T_U \ln \frac{V_B}{V_A}}$$

$$\eta = 1 - \frac{T_L}{T_U}$$

- **No heat engine operating between T_U and T_L can have an efficiency greater than a Carnot cycle efficiency.**

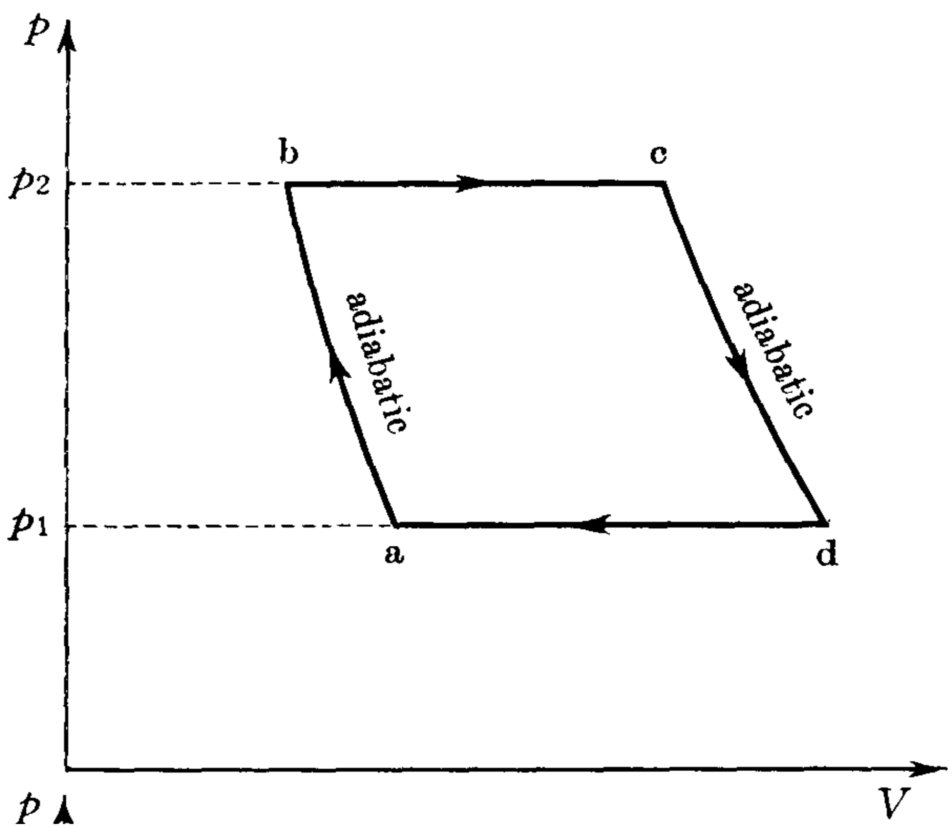
The Efficiency

- **Since T_L is constrained by the surroundings (ambient temperature for example), to achieve high efficiency, engineers will have to try to use the highest feasible operating temperature, T_U . T_U will be constrained by the materials the engine made of.**

Quiz IV

Show that the efficiency of the Joule cycle for an ideal gas is

$\eta = 1 - \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}}$. Assume that Cv, Cp and Cp/Cv $\equiv \gamma$ are constant.



Solution

Besides the work done in the adiabatic processes c→d and a→b, there is work done in the processes b→c and d→a. Hence

$$W = C_V\{(T_c - T_d) - (T_b - T_a)\} + p_2(V_c - V_b) - p_1(V_d - V_a).$$

Using $pV = nRT = (C_p - C_V)T = C_V(\gamma - 1)T$ we have

$$W = C_p\{T_c - T_b - T_d + T_a\}.$$

In the process b→c the gas receives heat $Q = C_p(T_c - T_b)$. For an adiabatic process $pV^\gamma = \text{const.}$, so that $Tp^{(1-\gamma)/\gamma} = \text{const.}$ and we have

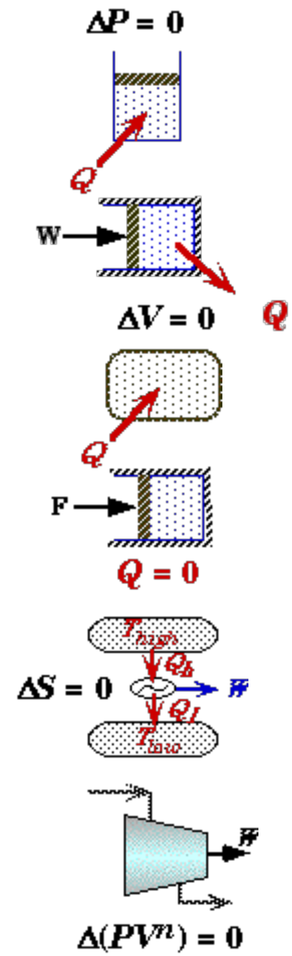
$$\frac{T_a}{T_b} = \left(\frac{p_1}{p_2}\right)^{(\gamma-1)/\gamma}, \quad \text{and} \quad \frac{T_d}{T_c} = \left(\frac{p_1}{p_2}\right)^{(\gamma-1)/\gamma}.$$

Therefore

$$\eta = \frac{W}{Q} = 1 - \frac{T_d - T_a}{T_c - T_b} = 1 - \left(\frac{p_1}{p_2}\right)^{(\gamma-1)/\gamma}.$$

Important Processes

- Isobaric (Pressure is const, $\Delta p = 0$)
- Isothermal (Temperature is Const, $\Delta T = 0$)
- Isochoric/Isovolumetric (Volume is const, $\Delta V = 0$)
- Adiabatic (No heat flows into or out of the system, Q = 0)
- Isoentropic (Entropy is const, $\Delta S = 0$)
- Polytropic (PV^n is const)



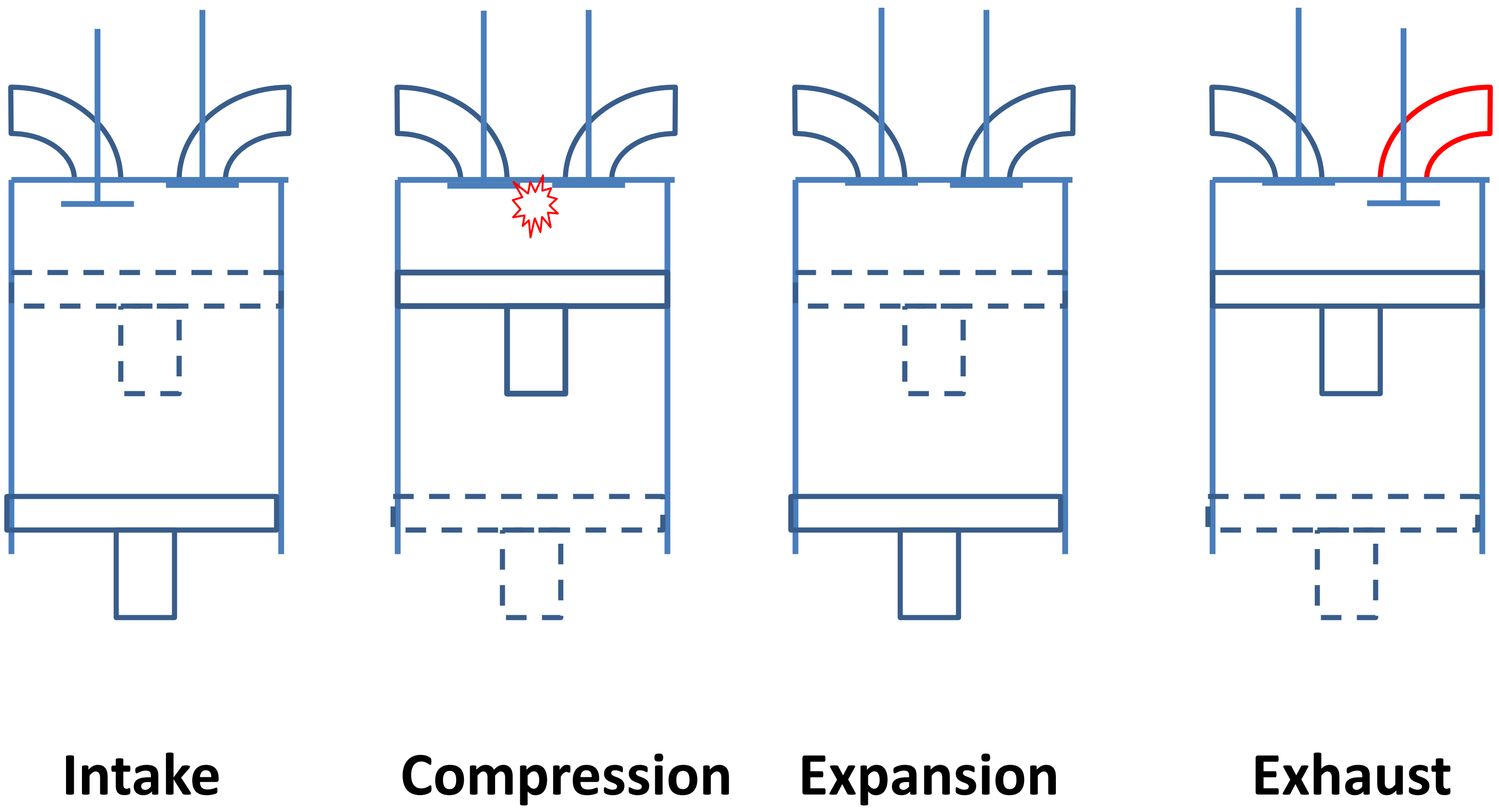
Important Cycles

- **The Lenoir cycle—**Patented by Jean Joseph Etienne Lenoir in 1860, is often thought of as the first commercially produced internal combustion engine
- In the cycle, an ideal gas undergoes
  - 1-2: Constant volume (isochoric) heat addition;
  - 2-3: Isentropic expansion;
  - 3-1: Constant pressure (isobaric) heat rejection—compression to the volume at the start of the cycle.

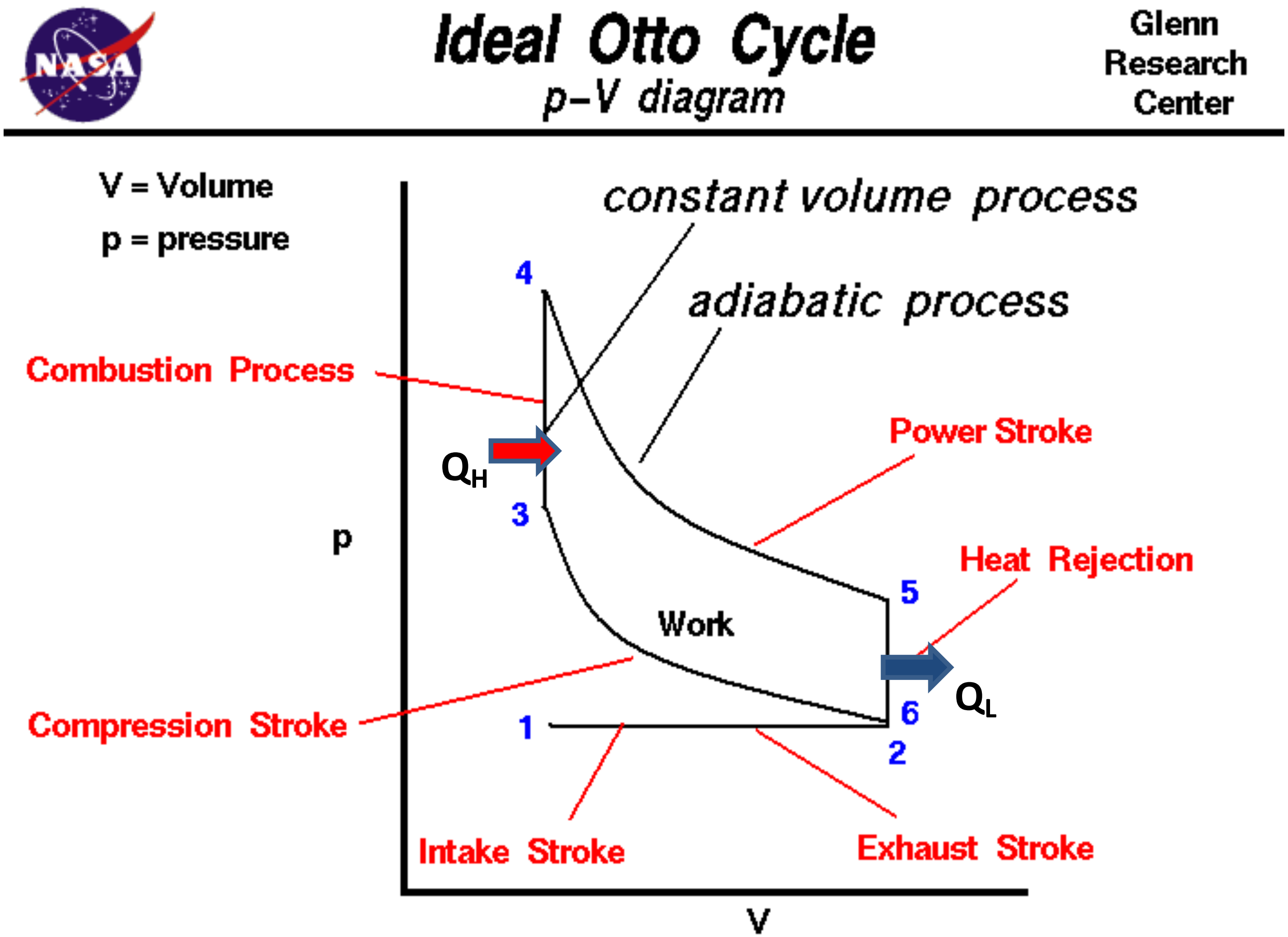
Important Cycles

- **The Otto cycle—**The four-stroke engine was first patented by Alphonse Beau de Rochas in 1861.
- The four strokes refer to intake, compression, combustion (power), and exhaust strokes that occur during two crankshaft rotations per working cycle of the gasoline engine.

The Otto cycle



The Otto cycle



The Otto cycle

Engine Thermodynamic Analysis

Ideal Otto Cycle

Glenn Research Center

C_v = Specific Heat constant volume

γ = Specific Heat Ratio

p = pressure

T = Temperature

V = Volume

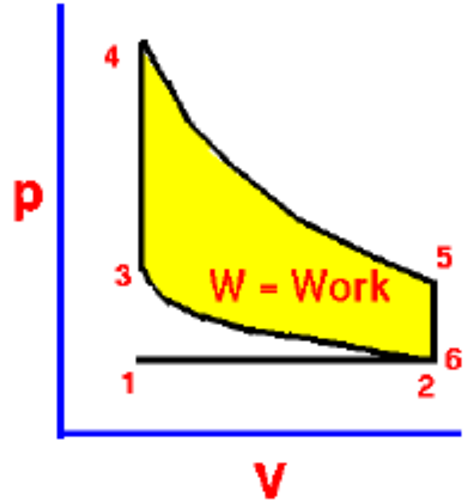
f = fuel / air ratio

Q = Fuel heating value

c p s = cycles per second

P = Power

$V_2 / V_3 = r$ = Compression Ratio



Compression Stroke:

$$p_3 / p_2 = r^{\gamma}$$

$$T_3 / T_2 = r^{\gamma - 1}$$

Combustion:

$$T_4 = T_3 + f\,Q / c_v$$

$$p_4 = p_3 \,(T_4 / T_3)$$

Power Stroke:

$$p_5 / p_4 = r^{-\gamma}$$

$$T_5 / T_4 = r^{1-\gamma}$$

Work per cycle:

$$W = C_v[(T_4 - T_5) - (T_3 - T_2)]$$

Engine Power:

$$P = W \ cps$$

cycles per second