Robust Control System

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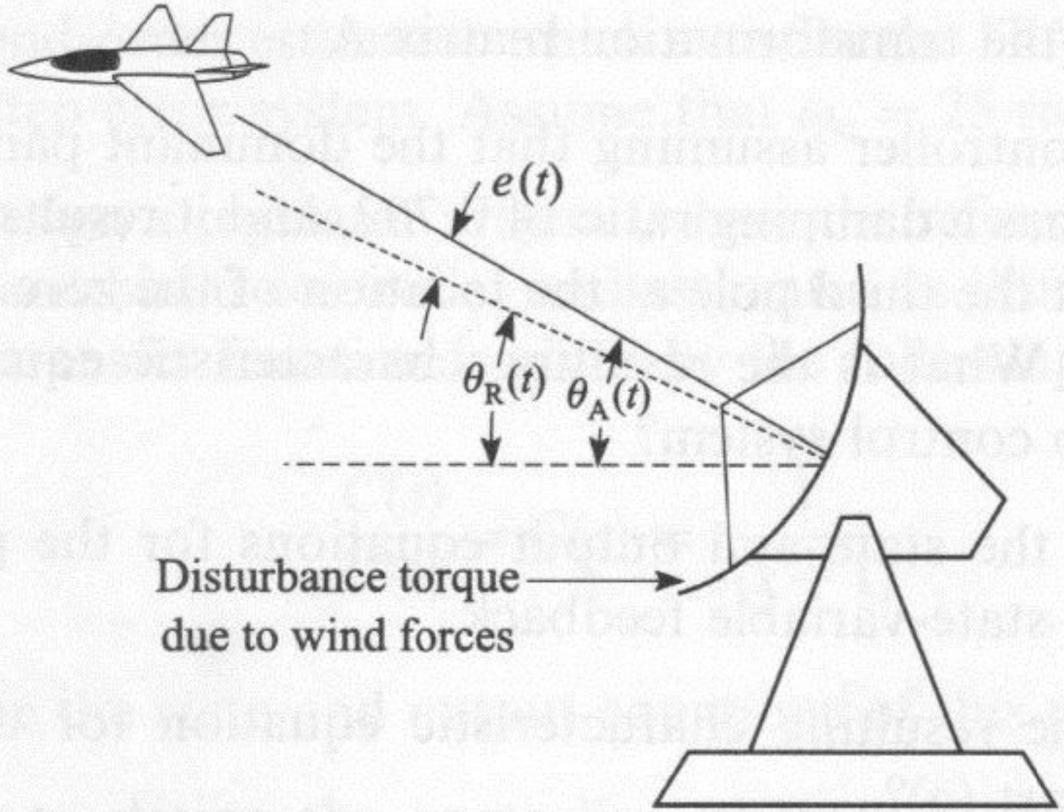
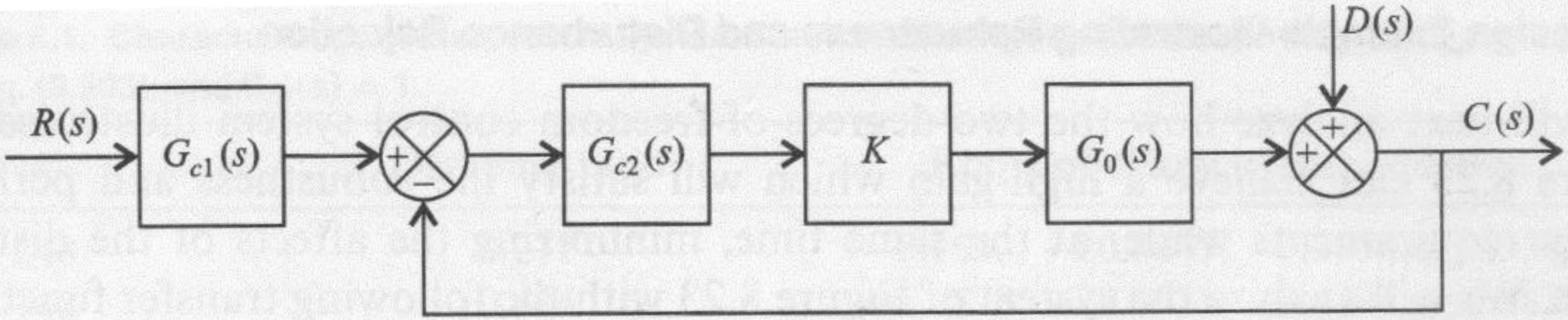

e(t)
θ_R(t)
θ_A(t)
Disturbance torque
due to wind forces


R(s)
G_c1(s)
+
−
G_c2(s)
K
G_0(s)
D(s)
+
+
C(s)

Quiz

Find $H(s) = \frac{C(s)}{R(s)}$ **,** $N(s) = \frac{C(s)}{D(s)}$ **and sensitivity** $S_K^{H(s)}$ **. What you have found ?**

$$H(s) = \frac{C(s)}{R(s)} = \frac{G_{c1}(s)G_{c2}(s)KG_0(s)}{1 + KG_{c2}(s)G_0(s)}$$

$$N(s) = \frac{C(s)}{D(s)} = \frac{1}{1 + KG_{c2}(s)G_0(s)}$$

$$S_K^{H(s)} = \frac{\dfrac{dH(s)}{H(s)}}{\dfrac{dK}{K}} = \frac{K}{H(s)}\frac{dH(s)}{dK}$$

$$= \frac{1}{1 + KG_{c2}(s)G_0(s)}$$

$$\frac{dH}{dK} = \frac{G_{c1}(s)G_{c2}(s)G_0(s)}{\left[1 + KG_{c2}(s)G_0(s)\right]^2}$$

Large K is good,
but how large can we have?
Need to consider stability !

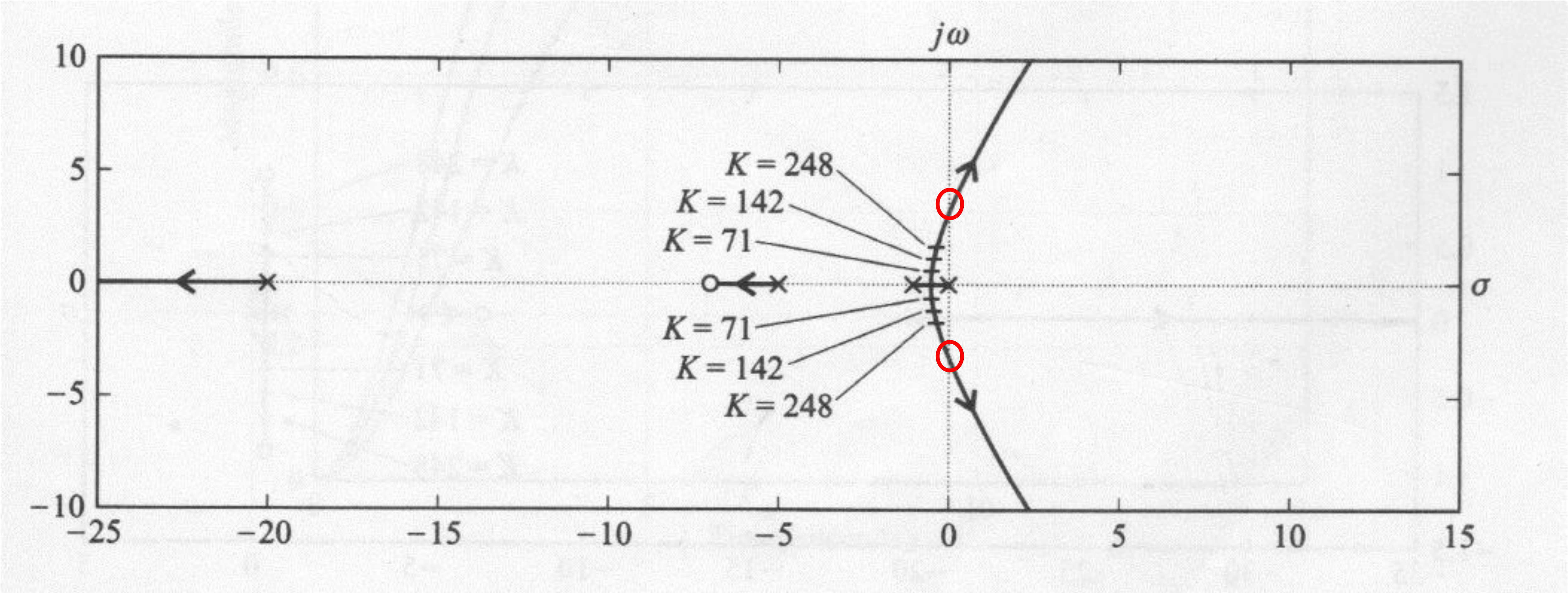
Quiz

Construct a Root-locus plot for the system

$$H(s) = \frac{C(s)}{R(s)} = \frac{G_{c1}(s)G_{c2}(s)KG_0(s)}{1 + KG_{c2}(s)G_0(s)}$$

Set G_{c1} and G_{c2} equal 1, and

$$KG_0(s) = \frac{K(7 + s)}{s(1 + s)(5 + s)(20 + s)}$$



How to find the complex conjugate roots with a given K?
Do it now for K = 142.

K	Damping ratio	Roots of characteristic equation
248	0.277	$-5.1640, -20.0649, -0.3856 \pm j1.3348$
142	0.444	$-5.0878, -20.0325, -0.4398 \pm j0.8875$
71	0.666	$-5.0456, -20.01629, -0.4691 \pm j0.5245$

- **If we place two zeros at (or near) the desired complex, conjugate-loop pools, for example -0.4398 $\pm$ j0.8875,**
- **set:**

$$G_{c2}(s) = \frac{(s + 0.4398 + j0.8875)(s + 0.4398 - j0.8875)}{0.98}$$

- **what will happen ?**

- **Simplify,**

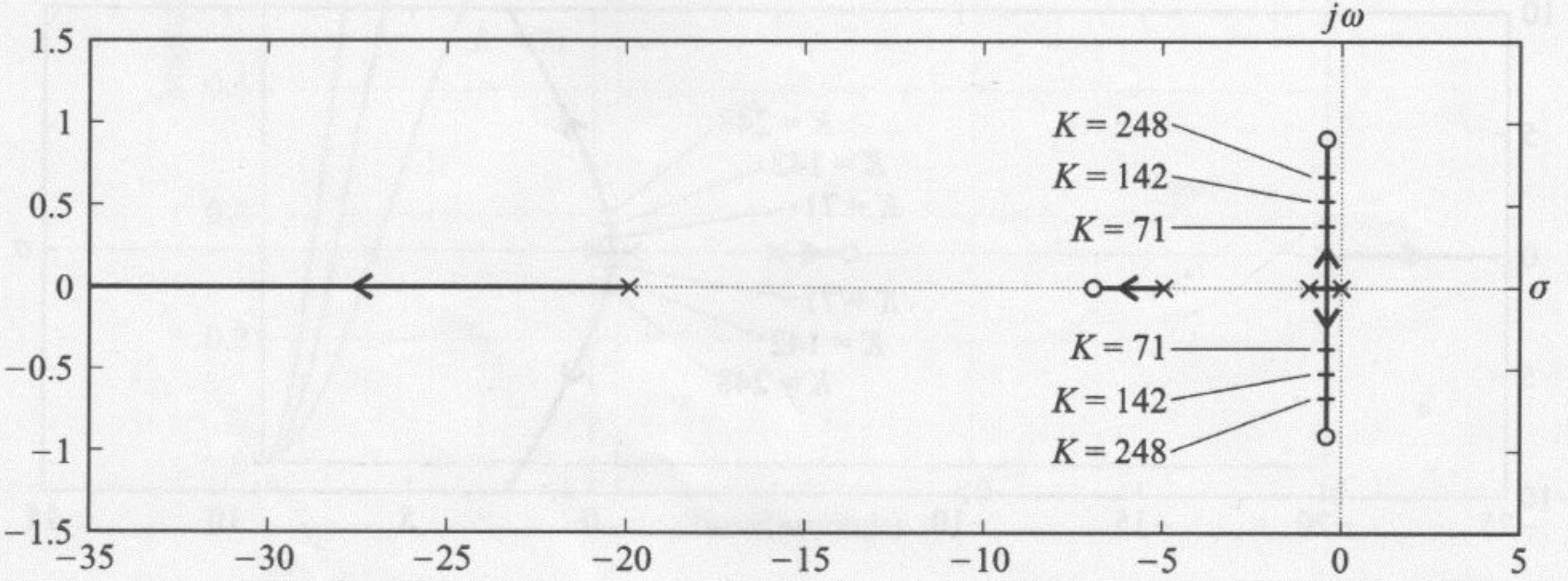
$$G_{c2}(s) = \frac{s^2 + 0.88s + 0.98}{0.98}$$

$$G_{c2}(s) = s^2 + 0.88s + 1$$

- **Keeping G_{c1}=1,**

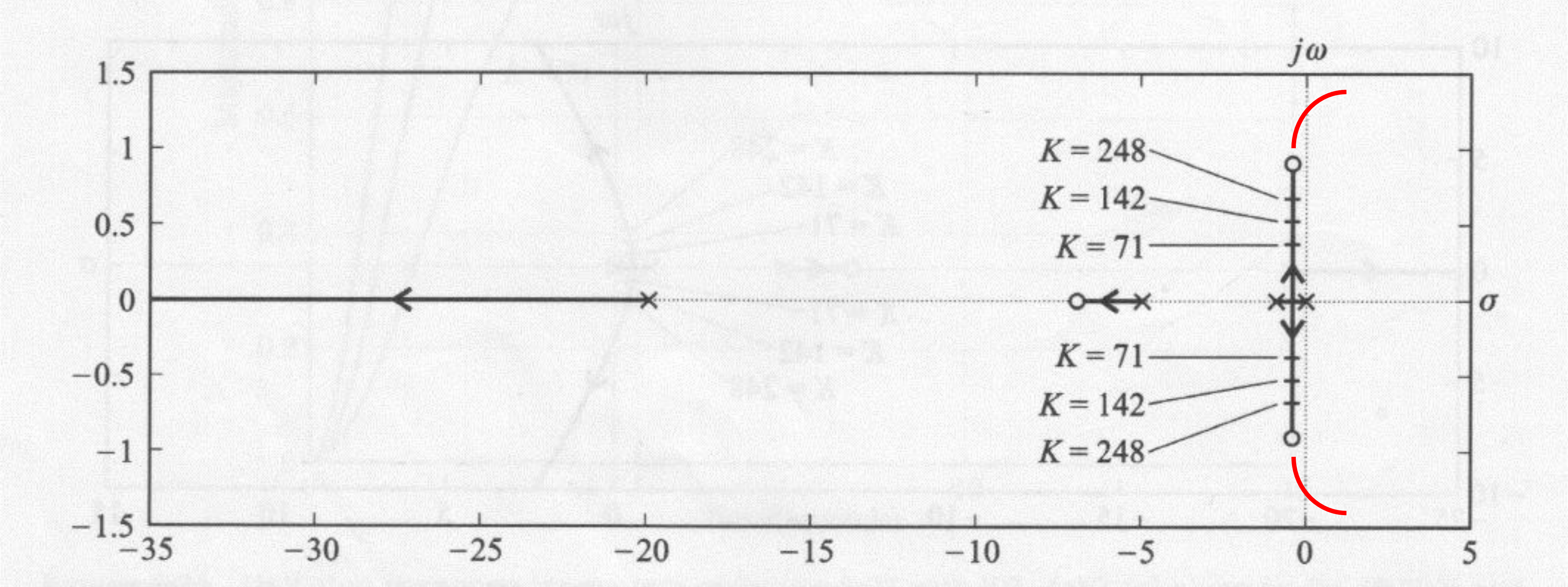
$$KG_{c2}(s)G_0(s) = \frac{K(7 + s)(s^2 + 0.88s + 1)}{s(1 + s)(5 + s)(20 + s)}$$

K	Damping ratio	Roots of characteristic equation
248	0.548	$-6.3715,\ -47.3367,\ -0.4459 \pm j0.6814$
142	0.644	$-6.0358,\ -33.3566,\ -0.4538 \pm j0.5392$
71	0.808	$-5.6914,\ -26.5282,\ -0.4652 \pm j0.3387$

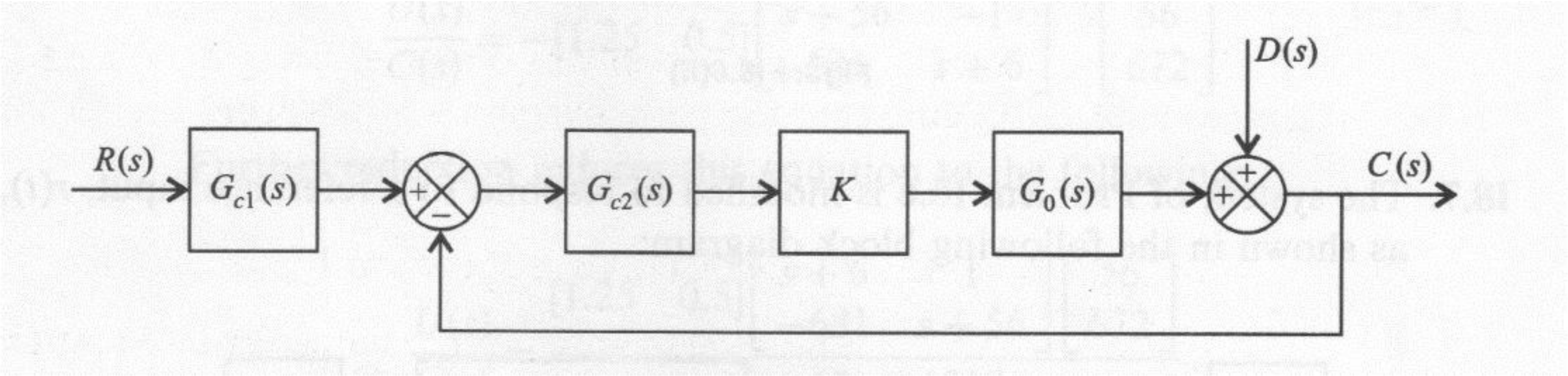


- **Not to change the characteristic of the system, by,**

- **Setting,** $$G_{c1}(s) = \frac{1}{s^2 + 0.88s + 1}$$



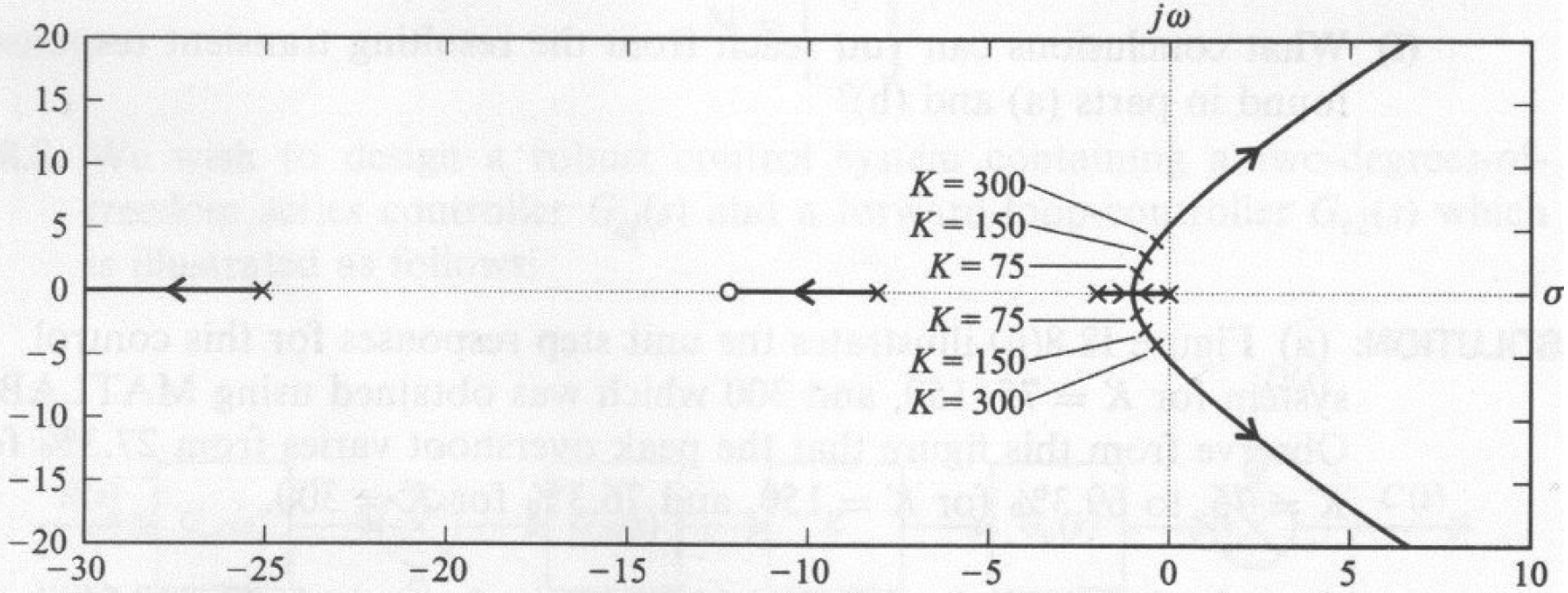
QUIZ



$$KG_0(s) = \frac{K(12 + s)}{s(2 + s)(8 + s)(25 + s)}$$

K = 75, 150 and 300

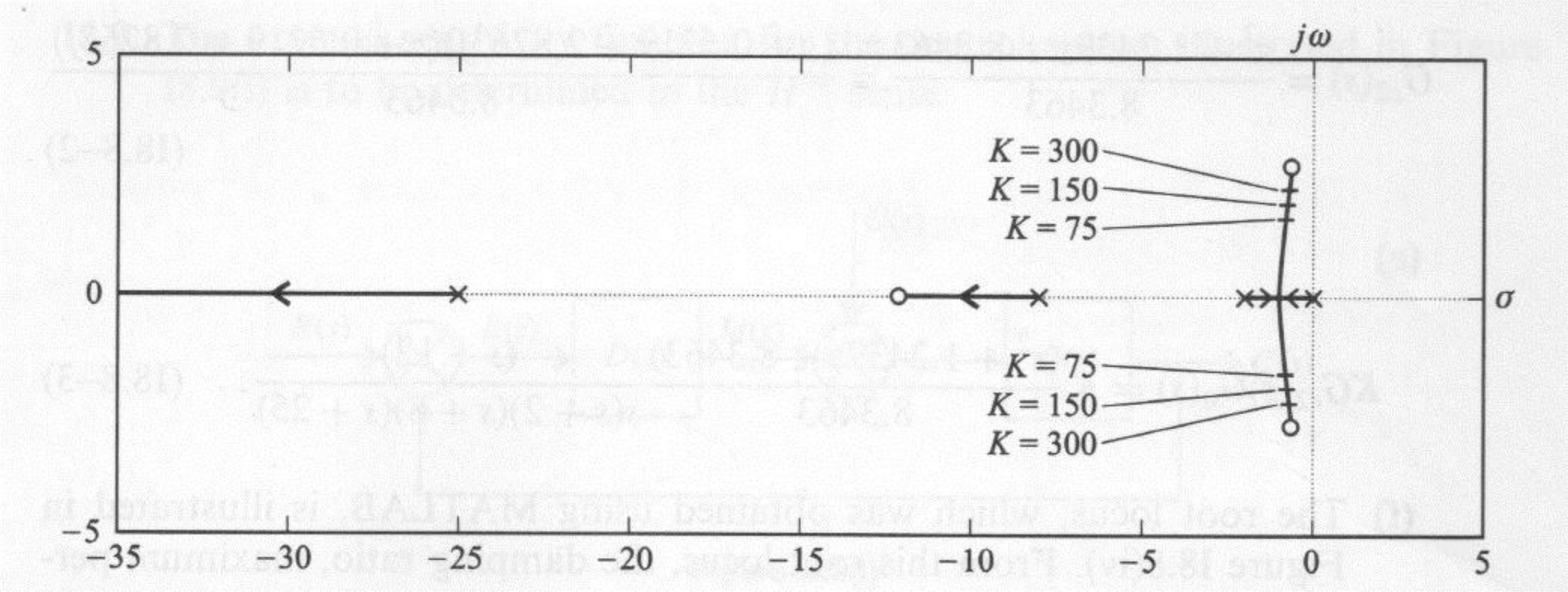
place zeros to cancel the poles of the system where k = 150



K	Damping ratio	Max % overshoot	Roots of characteristic equation
300	0.08577	76.3	$-8.9274, -25.3890, -0.3418 \pm j3.9707$
150	0.21528	69.3	$-8.5592, -25.1969, -0.6219 \pm j2.8213$
75	0.38176	27.3	$-8.3154, -25.0991, -0.7928 \pm j1.9193$

$$G_{c2}(s) = \frac{(s + 0.6219 + j2.8263)(s + 0.6219 - j2.8263)}{8.3463}$$

$$G_{c2}(s) = \frac{s^2 + 1.2439s + 8.3463}{8.3463}$$



$$KG_{c2}(s)G_0(s) = K\frac{(s^2 + 1.2439s + 8.3463)}{8.3463}\frac{(12 + s)}{s(2 + s)(8 + s)(25 + s)}$$

$$G_{c1}(s) = \frac{8.3463}{s^2 + 1.2439s + 8.3463}$$

Homework XII

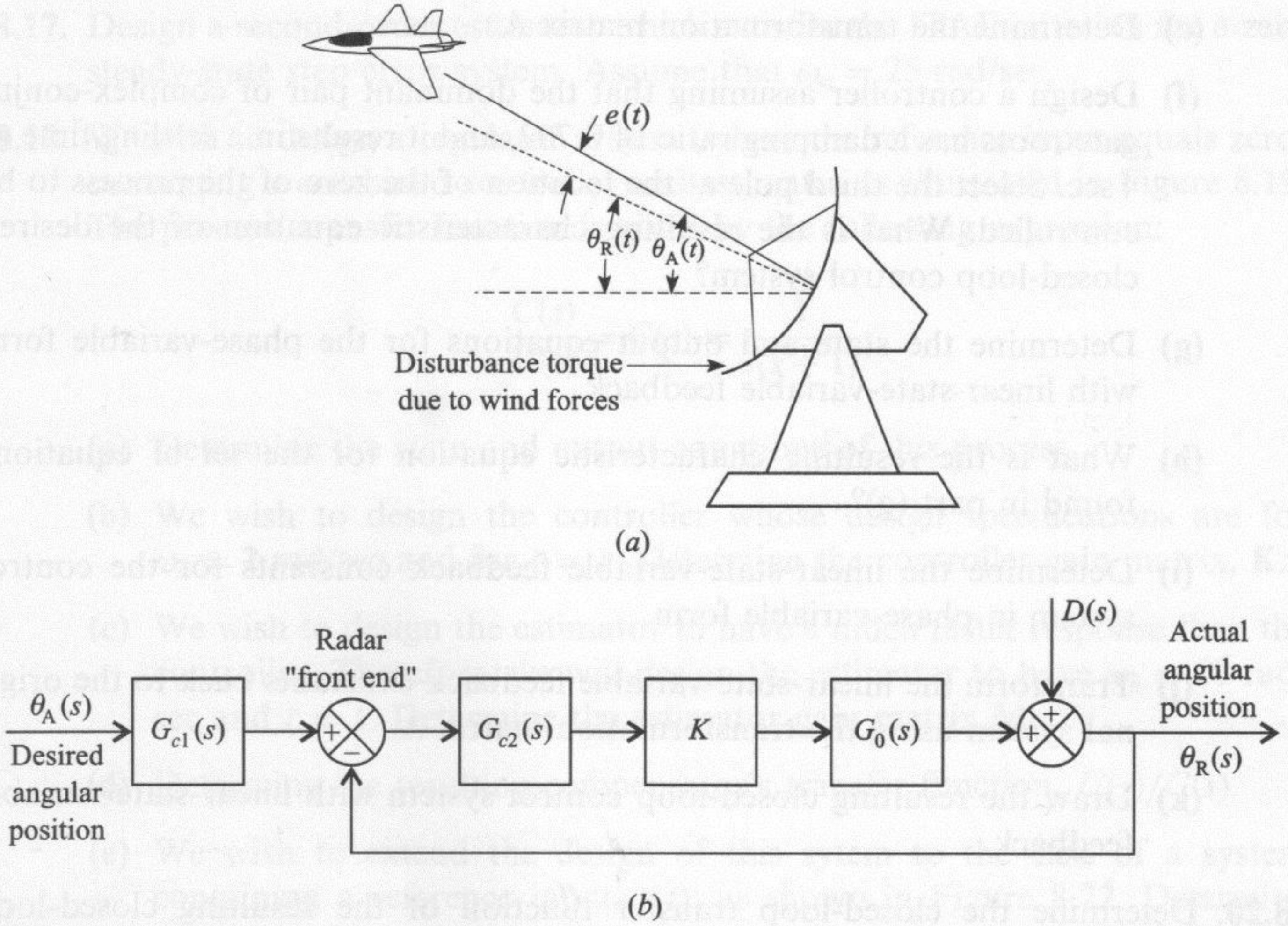


Figure P8.21 A tracking radar conceptually illustrating disturbance torques due to wind forces (*a*), and the equivalent block diagram of the tracking radar's positioning loop (*b*).

$$G_0(s) = \frac{(1 + 0.4s)}{s(1 + s)(1 + 0.15s)}$$

Choose K = 10, 40, 160

And use K = 40 for G_{c2}