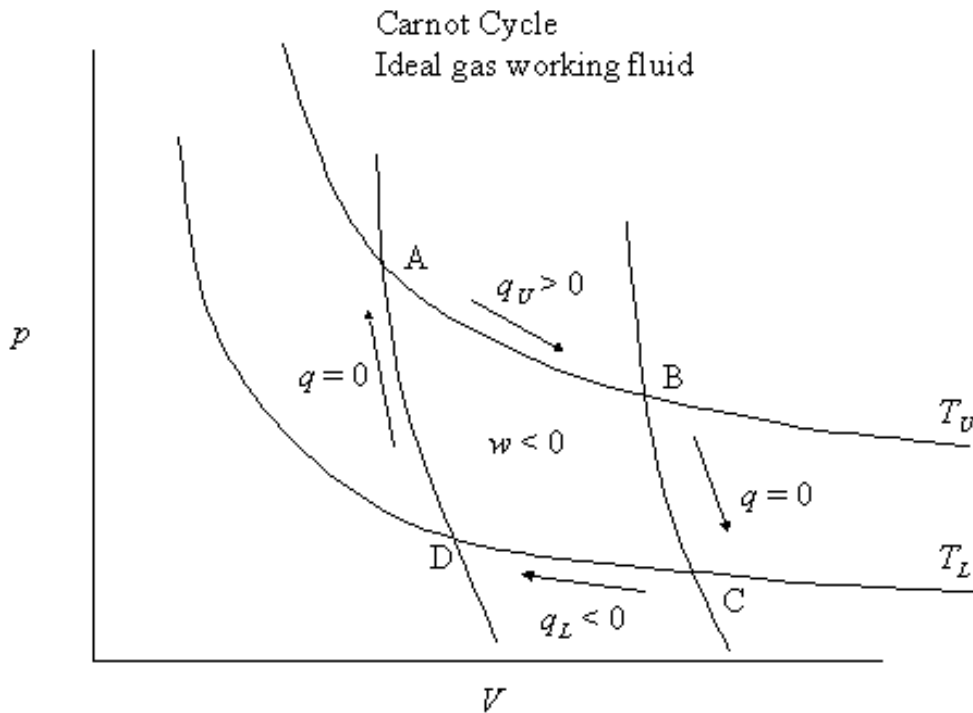


# Heat Engines and the Carnot Cycle

# Heat Engine and Cyclic Process

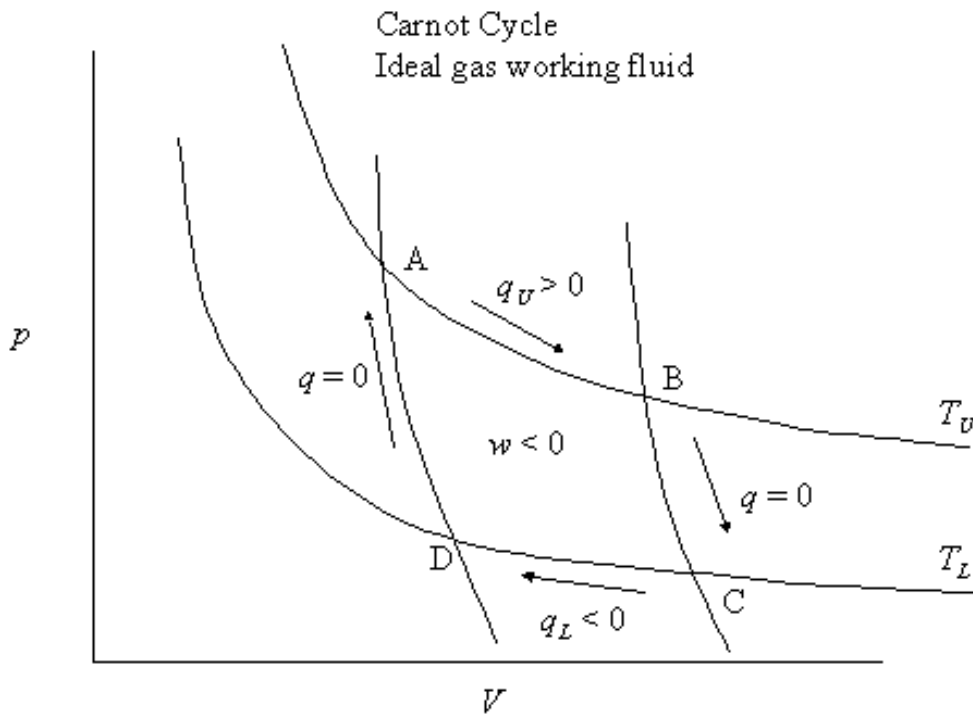
**A heat engine acts by transferring energy from a warm region to a cool region of space and, in the process, converting some of that thermal energy to mechanical work. "Cyclic" means that the system returns to its initial state at the end of each cycle so that there is no permanent change in the system.**

# Carnot Cycle



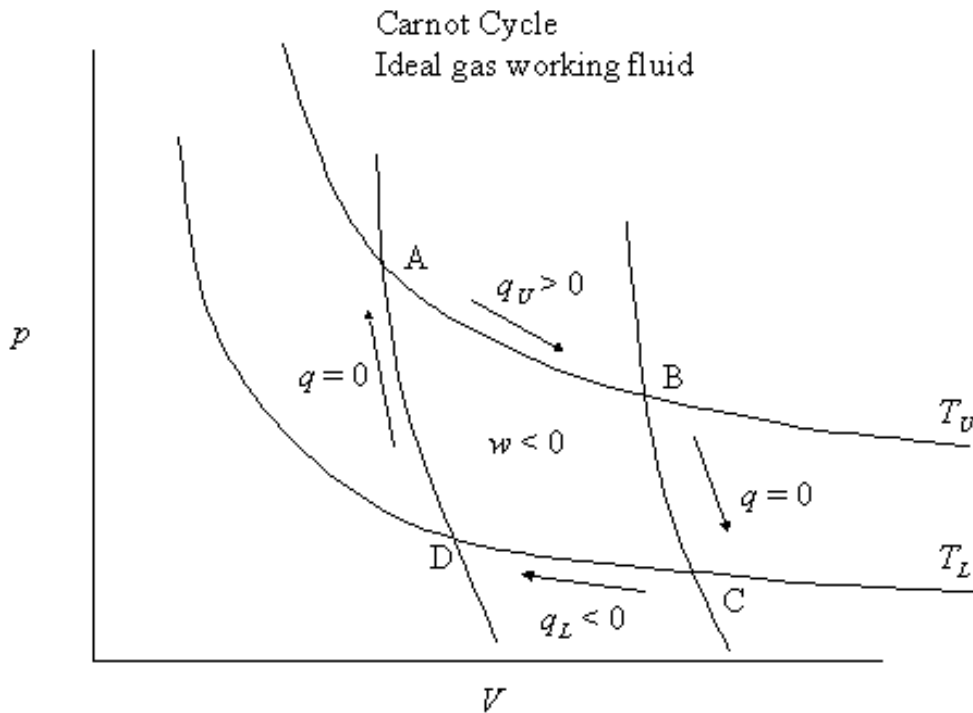
The curves labeled  $T_U$  and  $T_L$  are isotherms.  $T_U$  is the temperature of the upper temperature heat bath (reservoir) and  $T_L$  is the temperature of the lower temperature heat bath. The two steep curves, BC and DA, are adiabatic curves.

# Carnot Cycle



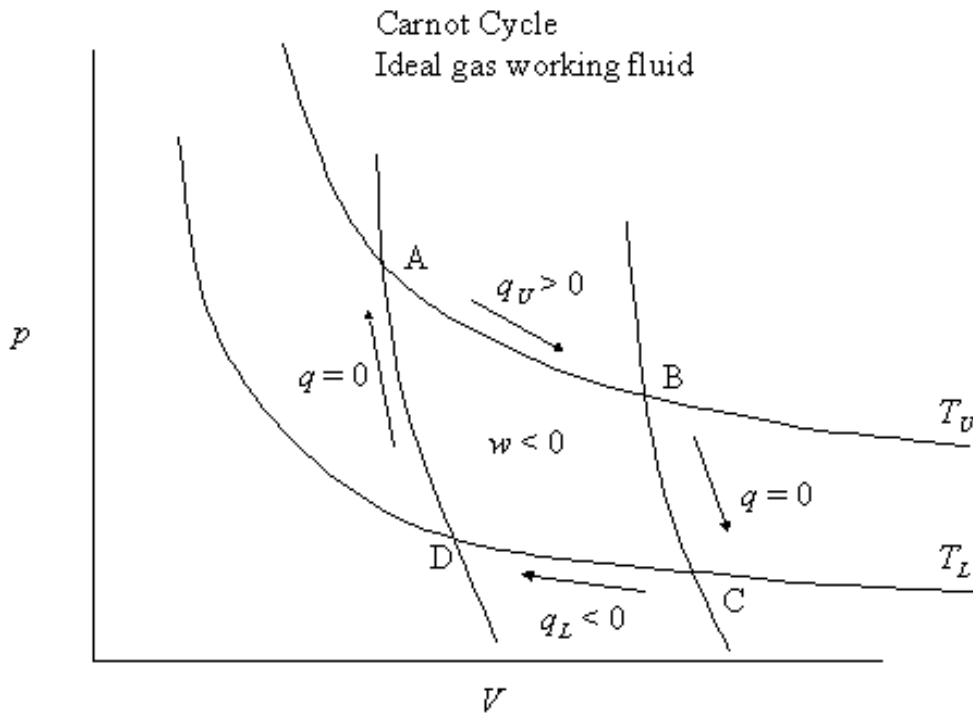
From point A, the system undergoes an isothermal expansion at temperature,  $T_U$ , to point B. In this isothermal expansion the system absorbs heat from the upper heat bath ( $q_U > 0$ ) and does work on the surroundings ( $w$  is defined as work done on the system so  $w_{AB} < 0$ ).

# Carnot Cycle



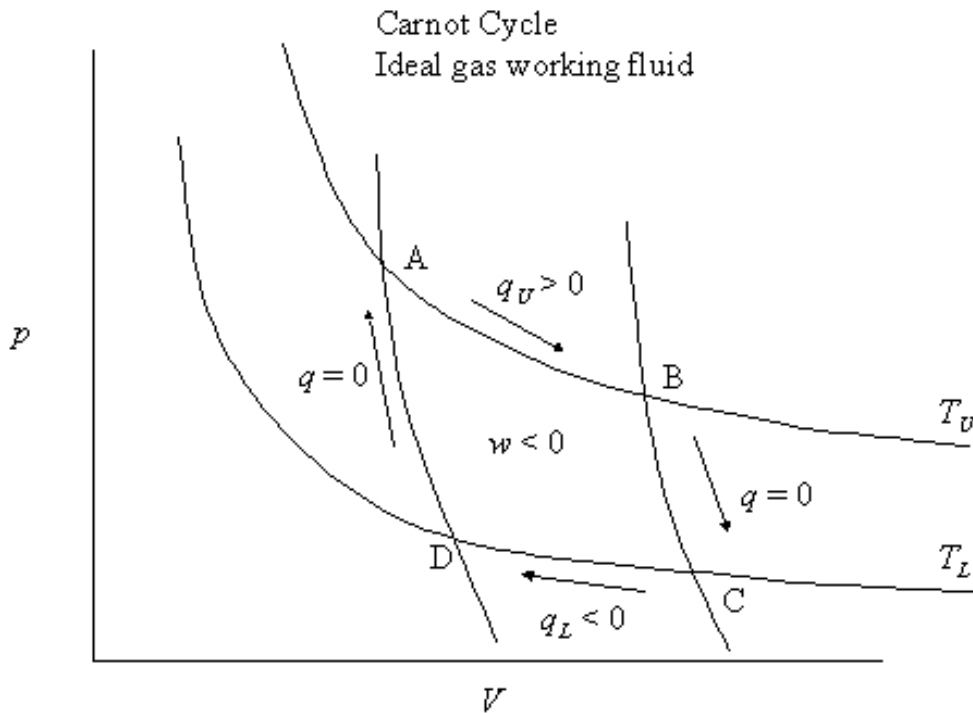
From point B, the system is then isolated from the heat bath and is expanded adiabatically to the point C. There is no heat in this adiabatic expansion, but the work for this step is also negative ( $w_{BC} < 0$ ).

# Carnot Cycle



- At point C the system is placed in contact with a heat bath at  $T_L$  and undergoes an isothermal compression to point D. For this segment of the cycle  $q_L < 0$  and  $w_{CD} > 0$  because the surroundings are now doing work on the system and heat is being dissipated to the heat bath at  $T_L$

# Carnot Cycle



At point D the system is again isolated from the heat baths and compressed adiabatically to point A. In this adiabatic compression the heat, of course, is zero and the work is positive ( $w_{DA} > 0$ )

The system completes a cycle.

# Carnot Cycle

- **The Carnot cycle is a particular thermodynamic cycle proposed by Nicolas Léonard Sadi Carnot in 1824 and expanded by Benoit Paul Émile Clapeyron in the 1830s and 40s.**
- **What makes the Carnot cycle special, is that it is the most efficient existing cycle converting a given amount of thermal energy into work.**



# The Efficiency

- The heat,  $q$ , for the whole cycle is

$$q = q_{AB} + q_{CD}$$

- the work for the entire cycle is,

$$W = W_{AB} + W_{BC} + W_{CD} + W_{DA}$$

- Since the initial state of the cycle is the same as the final state, the change in  $U$ , the internal energy, is zero.

$$\Delta U = q + w = 0$$

# The Efficiency

- Therefore,

$$q = -w$$

- The Carnot cycle is operating as a heat engine,  $w < 0$ , so that  $-w > 0$  and  $q > 0$ .
- As a heat engine, heat is absorbed and converted into work.
- the efficiency,  $\eta$ , of the cycle is

$$\eta = \frac{-w}{q_{AB}}$$

# The Efficiency

- Therefore,

$$\eta = \frac{q_{AB} + q_{CD}}{q_{AB}}$$

- the  $A \rightarrow B$  segment of the cycle is at the temperature of the upper heat bath,  $T_U$ , and the segment  $C \rightarrow D$  is at the temperature of the lower heat bath,  $T_L$ . Thus

$$\eta = \frac{q_U + q_L}{q_U} = 1 + \frac{q_L}{q_U}$$

# The Efficiency

- For isothermal expansions and compressions of ideal gases,
- $\Delta U = q + w = 0$ , therefore,

$$q_U = -w_{AB} + nRT_U \ln \frac{V_B}{V_A}$$

$$q_L = -w_{CD} + nRT_L \ln \frac{V_D}{V_C}$$

- where

$$w_{AB} = -\int_{V_A}^{V_B} p dV = -\int_{V_A}^{V_B} \frac{nRT}{V} dV = -nRT \ln \frac{V_B}{V_A}$$

# The Efficiency

- Therefore,

$$\eta = 1 + \frac{nRT_L \ln \frac{V_D}{V_C}}{nRT_U \ln \frac{V_B}{V_A}} = 1 + \frac{T_L \ln \frac{V_D}{V_C}}{T_U \ln \frac{V_B}{V_A}}$$

# Adiabatic Processes of Ideal Gas

- An adiabatic process is that heat transfer to the system is zero,  $\delta q = 0$
- From the first law and ideal gas,

$$dU + \delta w = \delta q = 0$$

$$\delta w = p dV$$

- From the Joule experiment,

$$dU = n C_v dT \text{ and } C_v = \alpha R \text{ (kinetic theory)}$$

$$dU = \alpha n R dT = \alpha d(PV) = \alpha (p dV + V dp)$$

# Adiabatic Processes of Ideal Gas

$$-pdV = \alpha pdV + \alpha Vdp$$

$$-(\alpha + 1)pdV = \alpha Vdp$$

$$-(\alpha + 1)\frac{dV}{V} = \alpha \frac{dp}{p}$$

$$\ln\left(\frac{p}{p_0}\right) = -\frac{\alpha + 1}{\alpha} \ln\left(\frac{V}{V_0}\right)$$

$$\left(\frac{p}{p_0}\right)\left(\frac{V}{V_0}\right)^{\frac{\alpha + 1}{\alpha}} = 1$$

$$pV^{\frac{\alpha + 1}{\alpha}} = p_0V_0^{\frac{\alpha + 1}{\alpha}} = pV^\gamma = \text{const.}$$

# The Efficiency

- For adiabatic paths  
B → C and D → A,

$$p_B V_B^\gamma = p_C V_C^\gamma$$

$$p_D V_D^\gamma = p_A V_A^\gamma$$

$$\frac{nRT_U}{V_B} V_B^\gamma = \frac{nRT_L}{V_C} V_C^\gamma$$

$$T_U V_B^{\gamma-1} = T_L V_C^{\gamma-1}$$

$$\frac{T_U}{T_L} = \frac{V_C^{\gamma-1}}{V_B^{\gamma-1}} \quad \text{and} \quad \frac{T_U}{T_L} = \frac{V_D^{\gamma-1}}{V_A^{\gamma-1}}$$

$$\frac{V_C^{\gamma-1}}{V_B^{\gamma-1}} = \frac{V_D^{\gamma-1}}{V_A^{\gamma-1}}$$

$$\frac{V_C}{V_B} = \frac{V_D}{V_A}$$



# The Efficiency

- Therefore,

$$\eta = 1 + \frac{nRT_L \ln \frac{V_D}{V_C}}{nRT_U \ln \frac{V_B}{V_A}} = 1 + \frac{T_L \ln \frac{V_A}{V_B}}{T_U \ln \frac{V_B}{V_A}}$$

$$\eta = 1 - \frac{T_L}{T_U}$$

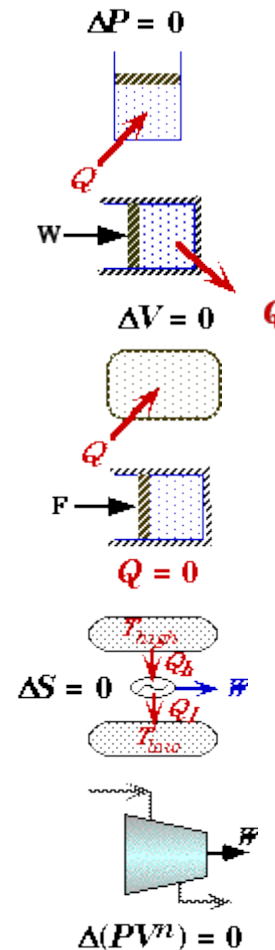
- No heat engine operating between  $T_U$  and  $T_L$  can have an efficiency greater than a Carnot cycle efficiency.

# The Efficiency

- Since  $T_L$  is constrained by the surroundings (ambient temperature for example), to achieve high efficiency, engineers will have to try to use the highest feasible operating temperature,  $T_U$ .  $T_U$  will be constrained by the materials the engine made of.

# Important Processes

- Isobaric (Pressure is const,  $\Delta p = 0$ )
- Isothermal (Temperature is Const,  $\Delta T = 0$ )
- Isochoric/Isovolumetric (Volume is const,  $\Delta V = 0$ )
- Adiabatic (No heat flows into or out of the system,  $Q = 0$ )
- Isoentropic (Entropy is const,  $\Delta S = 0$ )
- Polytropic ( $PV^n$  is const)



# Important Cycles

- **The Lenoir cycle**—Patented by Jean Joseph Etienne Lenoir in 1860, is often thought of as the first commercially produced internal combustion engine
- In the cycle, an ideal gas undergoes
  - 1-2: Constant volume (isochoric) heat addition;
  - 2-3: Isentropic expansion;
  - 3-1: Constant pressure (isobaric) heat rejection—compression to the volume at the start of the cycle.

# Important Cycles

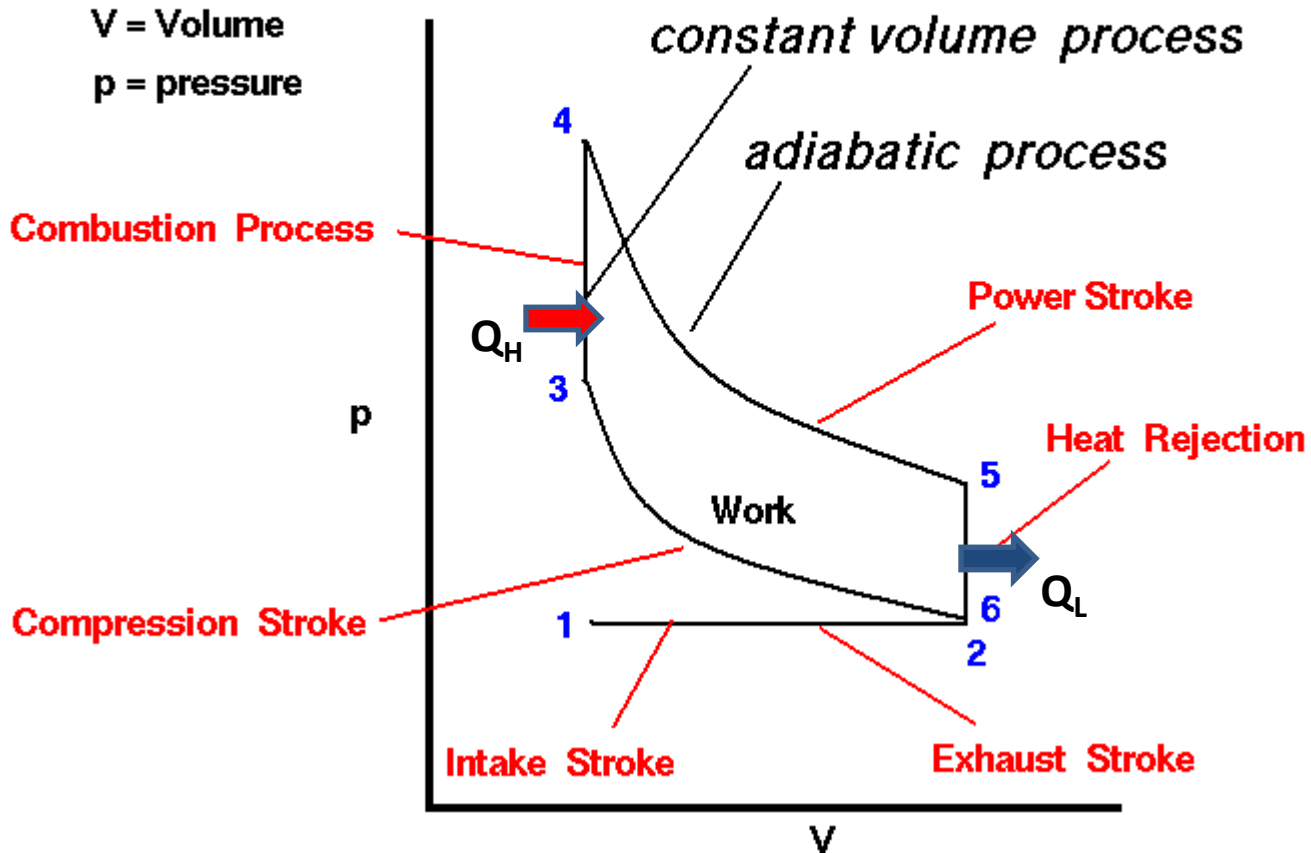
- **The Otto cycle**—The four-stroke engine was first patented by Alphonse Beau de Rochas in 1861.
- The four strokes refer to intake, compression, combustion (power), and exhaust strokes that occur during two crankshaft rotations per working cycle of the gasoline engine.

# The Otto cycle



## Ideal Otto Cycle *p-V diagram*

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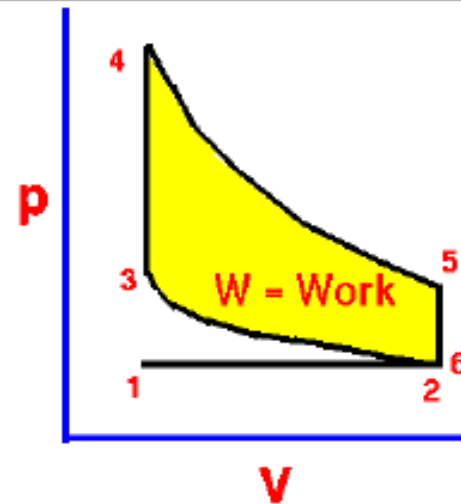
# The Otto cycle



## Engine Thermodynamic Analysis Ideal Otto Cycle

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- $C_v$  = Specific Heat constant volume
- $\gamma$  = Specific Heat Ratio
- $p$  = pressure
- $T$  = Temperature
- $V$  = Volume
- $f$  = fuel / air ratio
- $Q$  = Fuel heating value
- $cps$  = cycles per second
- $P$  = Power



$$V_2 / V_3 = r = \text{Compression Ratio}$$

Compression Stroke:

$$\begin{aligned} p_3 / p_2 &= r^\gamma \\ T_3 / T_2 &= r^{\gamma-1} \end{aligned}$$

Combustion:

$$\begin{aligned} T_4 &= T_3 + f Q / c_v \\ p_4 &= p_3 (T_4 / T_3) \end{aligned}$$

Power Stroke:

$$\begin{aligned} p_5 / p_4 &= r^{-\gamma} \\ T_5 / T_4 &= r^{1-\gamma} \end{aligned}$$

Work per cycle:

$$W = c_v [ (T_4 - T_3) - (T_5 - T_2) ]$$

Engine Power:

$$P = W \text{ cps}$$

cycles per second